



MATHEMATICS

Second Preparatory - Second Term

2025 - 2026

Student's Book

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Introduction

In the framework of the continuous development of the education system, and as part of the development plan that began with the kindergarten stage and continues until the end of the secondary stage, according to Egypt's Vision 2030 and the Ministry of Education's development plan, this book has been prepared to complete and implement the development processes of educational curriculums for the preparatory stage to create a qualitative leap in the way Egypt's students are prepared to deal with future challenges.

We are pleased to present this mathematics textbook for the second preparatory grade for the second term, which includes a set of tasks and performances that help students solve mathematical problems. This aims to achieve learning outcomes that keep pace with the challenges of the 21st century, whose structure is reinforced by the rapid revolution in information and communication technology.

The current curriculum aims to bring about a qualitative leap in the teaching and learning of mathematics. It also aims to provide an appropriate means to achieve the general educational goals in an integrated manner that aligns with the following:

- Emphasizing the positive role of the student in the learning process.
- Included knowledge curriculum, skills, values, positive trends needed for education, good citizenship, productive work and active participation in sustainable development programs.
- Including of modern positive trends in curriculum building, such as critical thinking skills, problem-solving skills, self-learning skills, collaborative learning, and effective communication with knowledge sources.
- Developing performance skills by focusing on self-learning and collaborative work.
- Achieving integration between mathematics and other subjects across different educational stages.
- Providing students with the opportunity to choose activities that suit their abilities, tendencies, and needs.

Asking Almighty God to make this book beneficial and hope that it will be part of a great effort to elevate Egypt to the ranks of advanced countries, ensuring a great future for all students. We wish you a very successful academic year full of achievements and success.

May God grant success,
The Authors

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UNIT 1

Numbers and Operations on them

Unit Lessons

1-1 Properties of Proportion

1-2 Continued Proportion



The concepts of proportion and continued proportion are considered among the mathematical foundations upon which the study of relative relationships between quantities is based. They appear clearly in artificial intelligence applications, as smart models depend on analyzing the proportional relationships between data, which helps us learn with accuracy/ precision and efficiency.

► **Can the concepts of Proportion and continued proportion be developed in activating Artificial Intelligence applications?**

Issues and Life Skills

- Mathematical Communication
- Critical Thinking
- Creative Thinking

Values

- Art Appreciation
- Respecting Others
- Environmental Awareness

Lesson 1 - 1

Properties of Proportion

■ Learning Outcomes

- Remember the concept of proportion.
- Understand the concepts of first, second, third, and fourth proportional.
- Understand the properties of proportion.
- Apply the properties of proportion in solving life problems.

■ Vocabulary

- Proportion
- First proportional
- Second proportional
- Third proportional
- Fourth proportional
- Extremes
- Means
- Cross multiplication

■ Previous Knowledge

The ratio between the two quantities a and b can be expressed in one of the following ways: a to b or $a : b$ or $\frac{a}{b}$ where a is called the antecedent of the ratio and b is called the consequent of the ratio.

Get Ready!



The tomb of King Tutankhamun has global fame, and it was discovered in 1922 A.D. A documentary channel produced a short film about King Tutankhamun and showed it on two digital platforms:

The film got 12,000 views during the first 4 hours of its screening on the first platform. While it got 18,000 views during the first 6 hours of its screening on the second platform.

Calculate the rate of views per hour for each platform. Are the two rates equal? If you add the number of views and the



**Golden Mask
of King Tutankhamun**

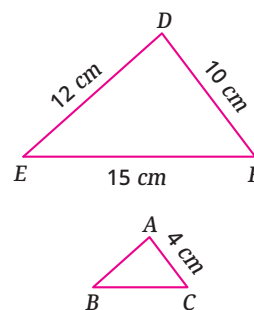
number of hours on both platforms together to calculate the rate of views per hour, would you get the same rate?

In this lesson, you will remember the concept of proportion, and learn about the properties of proportion, which will enable you to solve such problems.

Think & Discuss!



Triangle ABC is a reduction of triangle DEF by a certain ratio. Can you calculate the perimeter of triangle ABC ? Discuss.



Learn!



Proportion

Quantities a , b , c and d are said to be proportional if $\frac{a}{b} = \frac{c}{d}$

- a is called the first proportional • b is called the second proportional
- c is called the third proportional • d is called the fourth proportional
- Also, the terms a , d are called the extremes, and the terms b , c are called the means.

For Example:

► The numbers 4, 6, 10, 15 are proportional
because: $\frac{4}{6} = \frac{10}{15}$

► The numbers 2, 7, 4, 21 are not proportional
because: $\frac{2}{7} \neq \frac{4}{21}$

■ Note that

If $a \times d = b \times c$

Then $\frac{a}{b} = \frac{c}{d}$

■ Self-Evaluation ①

- ① Find the third proportional for the numbers 12, 28, 35
- ② If the quantities 21, 7a, 9a, 48 are proportional, find the value of a.

■ Self-Evaluation ②

If $\frac{2x+3}{2x-3} = \frac{2y+5}{2y-5}$

Then prove that $\frac{x}{y} = \frac{3}{5}$

Previous Knowledge: Cross multiplication property

If $\frac{a}{b} = \frac{c}{d}$ then: $a \times d = b \times c$

Meaning that: The product of extremes = The product of means.

Example 1

- ① Find the fourth proportional for the following numbers:

8, 12, 14



- ① Assuming the fourth proportional is X
 $\therefore$ The numbers 8, 12, 14, X are proportional.

$$\therefore \frac{8}{12} = \frac{14}{X}$$

$$\therefore 8 \times X = 12 \times 14$$

(Cross multiplication)

$$\therefore X = \frac{12 \times 14}{8} = 21$$

- ② Find the value of (a) that makes the following quantities:

a, 15, 20, 3a are proportional

- ② $\therefore$ The quantities a, 15, 20, 3a are proportional.

$$\therefore \frac{a}{15} = \frac{20}{3a}$$

$$\therefore a \times 3a = 15 \times 20$$

(Cross multiplication)

$$\therefore 3a^2 = 300$$

$$\therefore a^2 = \frac{300}{3} = 100$$

$$\therefore a = \pm \sqrt{100} = \pm 10$$

Example 2

If $\frac{3a+5b}{3a-5b} = \frac{3c+5d}{3c-5d}$

Prove that the quantities a, b, c, d are proportional.



$$\therefore \frac{3a+5b}{3a-5b} = \frac{3c+5d}{3c-5d}$$

$$\therefore (3a+5b)(3c-5d) = (3a-5b)(3c+5d) \text{ (Cross multiplication)}$$

$$\therefore \cancel{9ac} - 15ad + 15bc - \cancel{25bd} = \cancel{9ac} + 15ad - 15bc - \cancel{25bd}$$

$$\therefore -15ad + 15bc = 15ad - 15bc$$

$$\therefore 15bc + 15bc = 15ad + 15ad$$

$$\therefore 30bc = 30ad$$

$$\therefore bc = ad$$

$$\therefore \frac{a}{b} = \frac{c}{d}$$

$\therefore$ The quantities a, b, c, d are proportional.

Note

- If $\frac{a}{b} = \frac{c}{d} = k$ where k is a non-zero constant, k is called the constant of proportion, then by using cross multiplication: $a = bk$, $c = dk$
- In general: If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots = k$
then: $a = bk$, $c = dk$, $e = fk$, ...

■ Self-Evaluation ③

If a, b, c, d are proportional quantities, prove that:

$$\frac{4b+7d}{4a+7c} = \frac{5b-8d}{5a-8c}$$

■ Think & Discuss!

If $\frac{a}{b} = \frac{c}{d}$

Can you swap the extremes and get a proportion? Discuss.

■ Think with your classmates

How can you use Cross multiplication and properties of equality relation to prove the shown properties 1, and 2?

■ Self-Evaluation ④

If $\frac{2}{c} = \frac{3}{b} = \frac{4}{a}$

Prove that :

$$\frac{a+c}{c} \times \frac{b-c}{b} = 1$$

Example 3

If a, b, c, d are proportional quantities, prove that: $\frac{2a-3b}{a+5b} = \frac{2c-3d}{c+5d}$

✓ Assuming that:

$$\frac{a}{b} = \frac{c}{d} = k \quad (k \neq 0)$$

$$\therefore a = bk, \quad c = dk$$

$$\therefore \frac{2a-3b}{a+5b} = \frac{2bk-3b}{bk+5b} = \frac{\cancel{b}(2k-3)}{\cancel{b}(k+5)} = \frac{2k-3}{k+5} \quad (1)$$

$$\frac{2c-3d}{c+5d} = \frac{2dk-3d}{dk+5d} = \frac{\cancel{d}(2k-3)}{\cancel{d}(k+5)} = \frac{2k-3}{k+5} \quad (2)$$

From (1), (2) we conclude that: $\frac{2a-3b}{a+5b} = \frac{2c-3d}{c+5d}$

Properties of Proportion

1 If $\frac{a}{b} = \frac{c}{d}$ Then $\frac{b}{a} = \frac{d}{c}$ (Equality of inverse)

Example: $\frac{2}{4} = \frac{8}{16} (= \frac{1}{2})$ Then $\frac{4}{2} = \frac{16}{8} (= 2)$

2 If $\frac{a}{b} = \frac{c}{d}$ Then $\frac{a}{c} = \frac{b}{d}$ (interchange the means)

Example: $\frac{2}{4} = \frac{8}{16} (= \frac{1}{2})$ Then $\frac{2}{8} = \frac{4}{16} (= \frac{1}{4})$

Example 4

If $\frac{a}{b} = \frac{4}{3}$ What is the value of: $\frac{3a-2b}{3a+2b}$?

✓ $\therefore \frac{a}{b} = \frac{4}{3} \quad \therefore \frac{a}{4} = \frac{b}{3}$

Assuming that: $\frac{a}{4} = \frac{b}{3} = k$

$$\therefore a = 4k, \quad b = 3k$$

$$\begin{aligned} \therefore \frac{3a-2b}{3a+2b} &= \frac{3(4k)-2(3k)}{3(4k)+2(3k)} \\ &= \frac{12k-6k}{12k+6k} \\ &= \frac{6k}{18k} = \frac{1}{3} \end{aligned}$$

■ Diversified Strategies

Divide both numerator and denominator by b :

$$\begin{aligned} \therefore \frac{3a-2b}{3a+2b} &= \frac{3(\frac{a}{b})-2}{3(\frac{a}{b})+2} \\ \therefore \frac{a}{b} &= \frac{4}{3} \\ \therefore \frac{3a-2b}{3a+2b} &= \frac{3(\frac{4}{3})-2}{3(\frac{4}{3})+2} \\ &= \frac{4-2}{4+2} = \frac{2}{6} = \frac{1}{3} \end{aligned}$$

■ Self-Evaluation ⑤

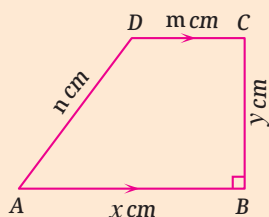
$$\text{If } \frac{a}{3} = \frac{b}{4} = \frac{c}{5}$$

$$= \frac{4a + mb + 5c}{25}$$

Find the value of m

■ Self-Evaluation ⑥

In the following figure:



$ABCD$ is a right-angled trapezium at B , its perimeter equals 36 cm ,
If $10x = 15y = 20m = 12n$
Find the value of each of x, y, m, n

3 If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots = k$, then :

① $\frac{a+c+e+\dots}{b+d+f+\dots} = k$ (in other words: $\frac{\text{Sum of antecedents}}{\text{Sum of consequents}} = \text{one of the ratios}$)

② $\frac{am+cn+el+\dots}{bm+dn+fl+\dots} = k$ where $m, n, \ell, \dots$ are non-zero real numbers

Example 5

If $\frac{x}{a} = \frac{y}{6} = \frac{4x-3y}{10}$, find the value of a

✓ $\therefore \frac{x}{a} = \frac{y}{6} = \frac{4x-3y}{10}$

By multiplying the terms of the first ratio by 4 and the second ratio by -3 :

$$\therefore \frac{4x}{4a} = \frac{-3y}{-18} = \frac{4x-3y}{10}$$

By adding the antecedents and consequents of the first and second ratios:

$$\therefore \frac{4x-3y}{4a-18} = \frac{4x-3y}{10}$$

$$\therefore 4a-18=10 \quad \therefore 4a=10+18 \quad \therefore 4a=28 \quad \therefore a=7$$

Example 6

If $2a = 3b = 4c$, $a+b+c = 52$, find the values of a, b, c

✓ $\therefore 2a = 3b = 4c$

$$\therefore \frac{2a}{12} = \frac{3b}{12} = \frac{4c}{12}$$

(Where 12 is the least common multiple for numbers 2, 3, 4)

$$\therefore \frac{a}{6} = \frac{b}{4} = \frac{c}{3}$$

$$\therefore \frac{a+b+c}{6+4+3} = \frac{a}{6} = \frac{b}{4} = \frac{c}{3}$$

$$\therefore \frac{52}{13} = \frac{a}{6} = \frac{b}{4} = \frac{c}{3}$$

$$\therefore \frac{a}{6} = \frac{b}{4} = \frac{c}{3} = 4$$

$$\therefore a = 6 \times 4 = 24$$

$$b = 4 \times 4 = 16$$

$$c = 3 \times 4 = 12$$

■ Diversified Strategies

$$\therefore 2a = 3b = 4c$$

$$\therefore \frac{2a}{12} = \frac{3b}{12} = \frac{4c}{12}$$

$$\therefore \frac{a}{6} = \frac{b}{4} = \frac{c}{3}$$

$$\text{assuming that : } \frac{a}{6} = \frac{b}{4} = \frac{c}{3} = k$$

$$\therefore a = 6k, b = 4k, c = 3k$$

$$\therefore 6k + 4k + 3k = 52$$

$$\therefore 13k = 52$$

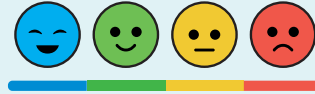
$$\therefore k = 4$$

$$\therefore a = 6 \times 4 = 24$$

$$b = 4 \times 4 = 16$$

$$c = 3 \times 4 = 12$$

Lesson Assessment



First Measuring Conceptual Understanding



► Choose the correct answer from the given ones:

① If $\frac{x}{6} = \frac{y}{10}$ What is the value of $\frac{x}{y}$?
 (a) $\frac{5}{3}$ (b) $\frac{5}{6}$ (c) $\frac{3}{5}$ (d) $\frac{5}{10}$

③ What is the second proportional for the numbers 4, 6, 12 ?
 (a) 18 (b) 8 (c) 6 (d) 2

⑤ If $\frac{a}{b} = \frac{c}{d} = \frac{2}{3}$, then $\frac{a+c}{b+d}$ equals :
 (a) $\frac{4}{3}$ (b) $\frac{2}{3}$ (c) $\frac{4}{9}$ (d) $\frac{1}{3}$

⑦ If $3a = 6b = 8c$, then $a : b : c$ equals:
 (a) 3 : 4 : 8 (b) 3 : 6 : 8
 (c) 8 : 4 : 3 (d) 8 : 6 : 3

⑨ If $\frac{a}{c+a-b} = \frac{b}{a+b-c} = \frac{c}{b+c-a}$
 then each ratio equals:
 (a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) 1

② If $\frac{a}{b} = \frac{2}{7}$ What is the value of $\frac{2b}{7a}$?
 (a) $\frac{4}{49}$ (b) $\frac{2}{7}$ (c) 1 (d) $\frac{49}{4}$

④ If $5x - 3y = 0$ What is the value of $\frac{x}{y}$?
 (a) $\frac{5}{3}$ (b) $\frac{3}{5}$ (c) $-\frac{3}{5}$ (d) $-\frac{5}{3}$

⑥ If $\frac{a}{5} = \frac{b}{3}$ What is the value of $\frac{a+b}{a-b}$?
 (a) $\frac{8}{3}$ (b) $\frac{5}{3}$ (c) $\frac{1}{4}$ (d) 4

⑧ If $\frac{a}{3} = \frac{b}{4}$, then what is the value of $\frac{3a-b}{b-a}$?
 (a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) $\frac{1}{5}$ (d) 5

⑩ If $m - n = 6$, $\frac{3}{m} = \frac{5}{n}$
 then what is the value of m ?
 (a) 15 (b) 9 (c) -9 (d) -15

Second Applying Concepts



⑪ Find the value of x that makes each of the following numbers proportional:

① 4, 5, x , 10

② x , 20, 30, 60

③ 2, 7, 5, $\frac{x}{2}$

④ $\frac{x}{2}$, $\frac{7}{x}$, $\frac{3x}{2}$, $\frac{7}{3}$

⑫ Find the number which if added to each of the numbers 7, 12, 11, 18 they become proportional numbers.

⑬ If $x : y = 2 : 5$,
 then find the value of
 $(10x + 3y) : (5x + 2y)$

⑭ If a, b, c, d are proportional quantities, prove each of the following:

① $\frac{a+b}{b} = \frac{c+d}{d}$

② $\frac{a-b}{b} = \frac{c-d}{d}$

③ $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

- ⑮ If $\frac{a-2b}{c-2d} = \frac{5a+b}{5c+d}$, prove that a, b, c, d are proportional quantities.

- ⑰ If $\frac{a+3b+3c}{3} = \frac{3a+4b-c}{4}$, find the value of $\frac{a}{c}$

- ⑲ If a, b, c are positive numbers such that $a:b:c = 2:3:4$ and $a^2 + b^2 + c^2 = 725$, find the value of $a + b - c$

- ⑯ If a, b, c, d are proportional quantities, prove that: $\frac{3a+4c}{3b+4d} = \frac{2a-c}{2b-d}$

- ⑱ If $\frac{x}{2} = \frac{y}{4} = \frac{z}{5}$ and $x + y - 2z = 12$, find the value of z

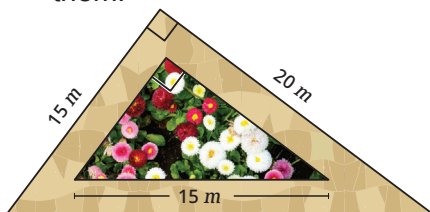
- ⑳ If $\frac{x+y}{7} = \frac{y+z}{5} = \frac{x+z}{8}$, find the value of $\frac{x+y+z}{x-z}$

Third

Analysis and Subjects Integration



- ⑳ **Computer Games:** In a computer game, Omar, Khaled, and Hamza scored 581 points together, such that four times Omar's points equals five times Khaled's points, and equals seven times Hamza's points. Find the points obtained by each of them.



- ㉒ **Landscaping:**

In the given figure:

A flower bed in the shape of a right-angled triangle inside a triangular piece of land whose side lengths are proportional to the corresponding side lengths in the flower bed. What are the lengths of the two legs of the right angle for the flower bed?

Creative Thinking

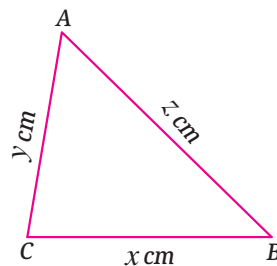


- ㉓ In the given figure:

ABC is a triangle whose perimeter is 61 cm, and its side lengths are x cm, y cm, z cm. If the side length x is reduced by 8 cm, the side length z is reduced by 4 cm, and the side length y is increased by 15 cm, these lengths become proportional to the numbers 3, 5, 8 respectively.

Calculate the length of $\overline{AB}$

- ㉔ If $\frac{x^2-9}{y+3} = \frac{y^2+2xy}{x} = 13$, find the value of $x + y$



Evaluate your understanding!

How well do you understand the properties of proportion? Tick the right box


☐
☐
☐
☐

Lesson 1 - 2

Continued Proportion

■ Learning Outcomes

- Understand the concept of continued proportion.
- Understand the concept of mean proportional between two numbers.
- Use the properties of continued proportion in solving problems.
- Use continued proportion in solving some life problems.

■ Vocabulary

- Continued proportion
- Mean proportional

■ Think

- ▶ Are the numbers 6, 9, 18 proportional?
- ▶ Are the numbers 3, 12, 48 proportional?

Get Ready!



The Egyptian Museum in Cairo is considered one of the largest Museums in the world. It offers visitors an exceptional opportunity to explore the ancient Egyptian civilization and Learn! about the daily life of ancient Egyptians. In an online advertising campaign to promote visiting the Egyptian Museum, the marketing team noticed that: The cost of the advertisement, the number of times it appeared, and the number of interactions are proportional.



The Egyptian Museum in Cairo

If the advertisement cost is 25,000 pounds, and the number of interactions is 1,440, what is the number of times the advertisement appeared?

In this lesson, you will Learn! the concept of continued proportion and its properties, which will enable you to solve such problems.

Think & Discuss!



Bassem has a ribbon 19 cm long, he wants to use it to make a triangle with proportional side lengths, such that the length of the longest side is 9 cm. What is the length of each of the other two sides?



Learn!



The continued proportion

If the quantities a , b , c are proportional, then: $\frac{a}{b} = \frac{b}{c}$

This proportion is called continued proportion

- a is called the first proportional.
- b is called the mean proportional between a , c
- c is called the third proportional.

For Example:

- The numbers 4, 6, 9 are in continued proportion because:

$$\frac{4}{6} = \frac{6}{9} \text{ (Note that each of the two ratios equals } \frac{2}{3} \text{)}$$

- The numbers 15, 12, 10 are not in continued proportion because: $\frac{15}{12} \neq \frac{12}{10}$

■ Enriched Information

If a, b, c are proportional numbers, then b is called the geometric mean between a, c .

■ Self-Evaluation ①

- ① Find the first proportional for the numbers 12, 9
- ② Find the mean proportional between the numbers 2, 8
- ③ Find the mean proportional between: $3x^3y^3, 27x^3y$.

■ Self-Evaluation ②

If Ibrahim has three sons aged 5 years, 7 years, 11 years now, How many years ago were their ages proportional?

The mean proportional

If b is the mean proportional between a, c , then: $\frac{a}{b} = \frac{b}{c}$
from cross multiplication, then:

$$b^2 = ac \text{ and hence } b = \pm \sqrt{ac}$$

Example 1

- ① Find the third proportional for the numbers 6, 18
- ② Find the mean proportional between the numbers 15, 135
- ③ Find the mean proportional between $3(x+y)^2, 12(x-y)^2$



- ① Assuming the third proportional is X

$\therefore 6, 18, X$ are proportional numbers

$$\therefore \frac{6}{18} = \frac{18}{X} \quad \therefore 6X = 18 \times 18 \quad \therefore X = \frac{18 \times 18}{6} = 54$$

So the third proportional for the numbers 6, 18 is 54

- ② Assuming the mean proportional between the numbers 15, 135 is X

$$\therefore X = \pm \sqrt{15 \times 135} = \pm 45$$

So the mean proportional between the numbers 15, 135 is ± 45

- ③ Assuming the mean proportional between $3(x+y)^2, 12(x-y)^2$ is b

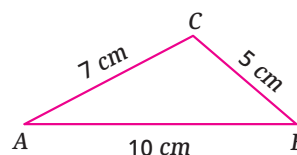
$$\begin{aligned} \therefore b &= \pm \sqrt{3(x+y)^2 \times 12(x-y)^2} = \pm \sqrt{36((x+y)(x-y))^2} \\ &= \pm \sqrt{36(x^2 - y^2)^2} = \pm 6(x^2 - y^2) \end{aligned}$$

So the mean proportional between

$3(x+y)^2, 12(x-y)^2$ is $\pm 6(x^2 - y^2)$

Example 2

What is the number of centimeters that must be subtracted from the length of each side of the opposite triangle to make its side lengths proportional?



Assuming the number of centimeters is X

$\therefore 5 - X, 7 - X, 10 - X$ are proportional numbers.

$$\therefore \frac{5-X}{7-X} = \frac{7-X}{10-X} \quad \therefore (7-X)^2 = (5-X)(10-X)$$

$$\therefore 49 - 14X + \cancel{X^2} = 50 - 15X + \cancel{X^2}$$

$$\therefore 15X - 14X = 50 - 49 \quad \therefore X = 1$$

$\therefore$ The number of centimeters that must be subtracted is 1 cm.

■ Self-Evaluation ③

If b is the mean proportional between a, c

Prove that :

$$\frac{c}{a} = \left(\frac{b-c}{a-b} \right)^2$$

■ Self-Evaluation ④

If $x, y, 27$ are in proportion, and $x - y = 30$

Find each of x, y

Note

If $\frac{a}{b} = \frac{b}{c} = k$ where k is a non-zero constant, then:

$$\textcircled{1} \quad b = ck$$

$$\textcircled{2} \quad a = bk = (ck)k$$

$$\therefore a = ck^2$$

Example 3

If b is the mean proportional between a, c

prove that: $\frac{3a+5b}{3b+5c} = \frac{2a-3b}{2b-3c}$



$\therefore b$ is the mean proportional between a, c

$$\therefore \frac{a}{b} = \frac{b}{c} = k \quad (k \neq 0)$$

$$\therefore b = ck, \quad a = ck^2$$

$$\begin{aligned} \therefore \frac{3a+5b}{3b+5c} &= \frac{3(ck^2)+5(ck)}{3(ck)+5c} \\ &= \frac{ck(3k+5)}{c(3k+5)} = k \end{aligned} \quad (1)$$

$$\begin{aligned} \frac{2a-3b}{2b-3c} &= \frac{2(ck^2)-3(ck)}{2(ck)-3c} \\ &= \frac{ck(2k-3)}{c(2k-3)} = k \end{aligned} \quad (2)$$

From (1), (2) we conclude that: $\frac{3a+5b}{3b+5c} = \frac{2a-3b}{2b-3c}$

■ Diversified strategies

$$\therefore \frac{a}{b} = \frac{b}{c} = k \quad (k \neq 0)$$

$$\therefore \frac{3a}{3b} = \frac{5b}{5c} = k$$

$$\therefore \frac{3a+5b}{3b+5c} = k \quad (1)$$

$$\frac{2a}{2b} = \frac{-3b}{-3c} = k$$

$$\therefore \frac{2a-3b}{2b-3c} = k \quad (2)$$

From (1), (2) we conclude

that: $\frac{3a+5b}{3b+5c} = \frac{2a-3b}{2b-3c}$

Example 4

If $x, y, 18$ are in proportion, $x + y = 20$ find each of x, y



$\therefore x, y, 18$ are in proportion.

$$\therefore \frac{x}{y} = \frac{y}{18}$$

$$\therefore \frac{x+y}{y+18} = \frac{y}{18} \quad (\text{Properties of Proportion})$$

$$\therefore \frac{20}{y+18} = \frac{y}{18}$$

$$\therefore y^2 + 18y = 360 \quad (\text{Cross multiplication})$$

$$\therefore y^2 + 18y - 360 = 0$$

$$\therefore (y-12)(y+30) = 0$$

$$\therefore y - 12 = 0$$

$$\text{or} \quad y + 30 = 0$$

$$\therefore y = 12$$

$$y = -30$$

$$\therefore x = 20 - 12$$

$$x = 20 + 30$$

$$\therefore x = 8$$

$$x = 50$$

Generalization of continued proportion

The quantities $a, b, c, d, \dots$ are said to be in continued proportion
If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \dots$

In this case: b is called the mean proportional between a, c
and c is called the mean proportional between b, d and so on...

For Example:

The numbers 24, 12, 6, 3 are in continued proportion.

Because: $\frac{24}{12} = \frac{12}{6} = \frac{6}{3}$ (Note that each ratio equals 2)

Notice that:

12 is a mean proportional between 24, 6

$$\text{where: } (12)^2 = 24 \times 6$$

6 is a mean proportional between 12, 3

$$\text{where: } (6)^2 = 12 \times 3$$

Note

If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = k$ where k is a non-zero constant, then:

$$\textcircled{1} \quad c = dk$$

$$\textcircled{2} \quad b = ck = (dk)k$$

$$\therefore b = dk^2$$

$$\textcircled{3} \quad a = bk = (dk^2)k$$

$$\therefore a = dk^3$$

■ Self-Evaluation ⑤

If a, b, c, d are in continued proportion, prove that:

$$\frac{c^2 - d^2}{a - c} = \frac{bd}{a}$$

■ Self-Evaluation ⑥

If a, b, c, d are in continued proportion, prove that:

$$\frac{b^2 - c^2}{c + d} = \frac{a^2 - bc}{a + b + c}$$

■ Note that

In Example ⑥

$$\begin{aligned} (dk^3)^2 &= dk^3 \times dk^3 \\ &= d^2 k^6 \end{aligned}$$

$$\begin{aligned} (dk^2)^2 &= dk^2 \times dk^2 \\ &= d^2 k^4 \end{aligned}$$

Example 5

If 3, $a, b, 24$ are in continued proportion, find the value of each of a, b



$\therefore 3, a, b, 24$ are in continued proportion.

$$\therefore \frac{3}{a} = \frac{a}{b} = \frac{b}{24} = k \quad (k \neq 0)$$

$$\therefore 3 = 24k^3 \quad \therefore k^3 = \frac{3}{24} = \frac{1}{8}$$

$$\therefore k = \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$$

$$\therefore a = 24k^2 = 24 \times \left(\frac{1}{2}\right)^2 = 6, \quad b = 24k = 24 \times \frac{1}{2} = 12$$

Example 6

If a, b, c, d are in continued proportion, prove that: $\frac{a^2 + c^2}{b^2 + d^2} = \frac{ac}{bd}$



$\therefore a, b, c, d$ in continued proportion.

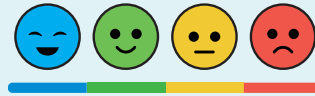
$$\therefore \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = k, \quad (k \neq 0) \quad \therefore c = dk, b = dk^2, a = dk^3$$

$$\therefore \frac{a^2 + c^2}{b^2 + d^2} = \frac{(dk^3)^2 + (dk)^2}{(dk^2)^2 + d^2} = \frac{d^2 k^6 + d^2 k^2}{d^2 k^4 + d^2} = \frac{\cancel{d^2} k^2 (\cancel{k^4} + 1)}{\cancel{d^2} (\cancel{k^4} + 1)} = k^2 \quad (1)$$

$$\frac{ac}{bd} = \frac{dk^3 \times dk}{dk^2 \times d} = \frac{\cancel{d^2} k^4}{\cancel{d^2} k^2} = k^2 \quad (2)$$

From (1), (2) we deduce that $\frac{a^2 + c^2}{b^2 + d^2} = \frac{ac}{bd}$

Lesson Assessment



First

Measuring Conceptual Understanding



► Choose the correct answer from the given ones:

- ① Which of the following numbers are proportional ?
 (a) 2, 6, 4 (b) 3, 5, 8
 (c) 3, 9, 27 (d) 1, 4, 9
- ③ If b is a mean proportional between 3, 12 What is the value of b ?
 (a) ± 6 (b) ± 6
 (c) ± 9 (d) ± 36
- ⑤ What is the third proportional for the numbers $X^3, 2X^2$?
 (a) $2X$ (b) $4X$
 (c) X^2 (d) $2X^5$
- ⑦ If $\frac{a}{b} = \frac{b}{c} = 3$, then which of the following is correct?
 (a) $a = 3c$ (b) $a = 9b$
 (c) $a = 9c$ (d) $a = 6c$
- ⑨ If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = 3$, then what is the value of $\frac{a+b+c}{b+c+d}$?
 (a) 1 (b) 3 (c) 9 (d) 6
- ② If a is the first proportional for the numbers 4, 16. What is the value of a ?
 (a) 8 (b) 4
 (c) 2 (d) 1
- ④ If X is the positive mean proportional between 24, 54. What is the value of X ?
 (a) 15 (b) 36
 (c) 39 (d) 78
- ⑥ If b is a mean proportional between $(12 - \sqrt{32}), (3 + \sqrt{2})$. What is the value of b ?
 (a) $\pm \sqrt{7}$ (b) $\pm 2\sqrt{7}$
 (c) $\pm 4\sqrt{7}$ (d) $\pm 16\sqrt{7}$
- ⑧ If $\frac{a}{b} = \frac{b}{c} = \frac{1}{2}$, then which of the following is correct?
 (a) $a = 2c$ (b) $c = 2a$
 (c) $a = 4c$ (d) $c = 4a$
- ⑩ If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = 4$, then what is the value of $\frac{a+d}{d}$?
 (a) 65 (b) 49 (c) 37 (d) 25

► Spot the mistake:

- ⑪ Both Mostafa and Mohamed solved the following problem:

If b is a mean proportional between a, c prove that: $\frac{3a+4b}{3b+4c} = \frac{a}{b}$

Mohamed's solution:

$\because b$ is the mean proportional between a, c

$$\therefore \frac{a}{b} = \frac{b}{c} = k$$

$$\therefore a = ck^2, b = ck$$

$$\therefore \frac{3a+4b}{3b+4c} = \frac{3ck^2+4ck}{3ck+4c}$$

$$= \frac{ck(3k+4)}{c(3k+4)} = k$$

$$\therefore \frac{3a+4b}{3b+4c} = \frac{a}{b}$$

Mostafa's solution:

$\because b$ is the mean proportional between a, c

$$\therefore \frac{a}{b} = \frac{b}{c} = k$$

$$\therefore \frac{3a}{3b} = \frac{4b}{4c} = k$$

$$\therefore \frac{3a+4b}{3b+4c} = k$$

$$\therefore \frac{3a+4b}{3b+4c} = \frac{a}{b}$$

Which of them followed the correct method in the solution? Discuss

Second

Applying Concepts



► Choose the correct answer from the given ones:

- ⑫ If $\frac{a}{b} = \frac{b}{c}$, then $\frac{a}{c}$ equals:
 (a) b^2 (b) $\frac{a+b}{b+c}$
 (c) $\frac{a^2}{b^2}$ (d) $\frac{ac}{b^2}$
- ⑬ If $\frac{a}{b} = \frac{b}{4}$, $a - b = -1$, then what is the value of $a \times b$?
 (a) 1 (b) 2
 (c) 4 (d) 8
- ⑭ If $\frac{a}{b} = \frac{b}{c} = \frac{4}{5}$ and $a + b + c = 122$, then what is the value of a ?
 (a) 4 (b) 32 (c) 40 (d) 50

► Answer the following questions:

- ⑮ In each of the following, if the given numbers are in proportion, find the value of X :
 ① 25, 75, X ② X , 24, 144 ③ 15, X , 60
- ⑯ In each of the following, find the value of each of X and Y such that the quantities are in continued proportion:
 ① X , 6, 12, Y ② 2, X , Y , 54
- ⑰ What is the number which if added to each of the numbers 3, 8, 18 they become proportional numbers?
- ⑱ If b is the mean proportional between a , c , then prove that:
 ① $\frac{a-c}{a+b} = \frac{a-2b+c}{a-b}$ ② $\frac{4a^2-9b^2}{4b^2-9c^2} = \frac{a}{c}$
- ⑲ If a, b, c, d are in continued proportion, then prove that: $\frac{a^3+b^3+c^3}{b^3+c^3+d^3} = \frac{a}{d}$
- ⑳ If a, b, c, d are in continued proportion, then prove that: $(b+c)$ is a mean proportional between $(a+b)$, $(c+d)$
- ㉑ If $\frac{3a-4b}{3b-4c} = \frac{b-a}{c-b}$, then prove that b is the mean proportional between a, c
- ㉒ If the numbers 3, X, Y are proportional, $X+Y=18$, then find the value of each of X and Y

Third

Analysis and Subjects Integration



- ㉓ National Values: A memorial flag was designed in the shape of a triangle used for exchange between teams in football matches as shown in the figure, such that the area of the upper part, the area of the middle part, and the area of the lower part are proportional. If the area of the middle part is 80 cm^2 , while the area of the upper part is 200 cm^2 , what is the area of the flag?



Creative Thinking



► Choose the correct answer from the given ones:

- ㉔ If $a : b = b : c$, then $a^4 : b^4$ equals
 (a) $ac : b^2$ (b) $a^2 : c^2$ (c) $c^2 : a^2$ (d) $b^2 : ac$
- ㉕ If b is the mean proportional between a, c , then $(a-b)^3 : (b-c)^3$ equals
 (a) $a^3 : c^3$ (b) $a^2 : c^2$ (c) $a^3 : b^3$ (d) $b^2 : c^2$

Evaluate your understanding!

How well do you understand continued proportion? Tick the right box



Unit One Assessment

Evaluate your understanding!

How well do you understand Unit One?
Tick the right box



► Choose the correct answer from the given ones:

- ① If $a : b = c : d$, then $\frac{2a+5c}{2b+5d}$ does not equal :
 (a) $\frac{a}{b}$ (b) $\frac{c}{d}$
 (c) $\frac{a+c}{b+d}$ (d) $\frac{a-c}{d-b}$
- ② If $3X = 5Y = 4Z$, then $X : Y : Z$ equals
 (a) $9 : 12 : 16$ (b) $20 : 12 : 15$
 (c) $15 : 10 : 9$ (d) $8 : 5 : 3$
- ③ If $\frac{a}{b} = \frac{b}{c} = 3$, then what is the value of $\frac{2a+3b}{4b-3c}$?
 (a) 1 (b) 2
 (c) 3 (d) $\frac{2}{3}$
- ④ What is the third proportional for the two quantities $X^2 - Y^2$ and $X - Y$?
 (a) $X + Y$ (b) $X - Y$
 (c) $\frac{X+Y}{X-Y}$ (d) $\frac{X-Y}{X+Y}$

► Complete each of the following with the correct answer:

- ⑤ If the numbers $a, b, 32$ are proportional,
 $a + b = 10$, then $(a, b) = \dots\dots\dots$
- ⑥ If the ratio among the side lengths of a triangle is $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$ and its perimeter equals 104 cm, then the length of the longest side in the triangle equals cm
- ⑦ If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = 2$, then $\frac{a+d}{d} = \dots\dots\dots$
- ⑧ The mean proportional between the numbers 3 and 48 equals

► Answer the following questions:

- ⑨ If a, b, c, d are in continued proportion, then prove that : $\frac{a-b}{a-2b+c} = \frac{a+b+c}{a-d}$
- ⑩ If the quantities : 35, 15, 3, $2X + 1$ are proportional, then find the numerical value of X
- ⑪ If $\frac{a+b}{25} = \frac{a-b}{11} = \frac{a+b-c}{8}$, then find the ratio $a : b : c$
- ⑫ Find the number which if subtracted from each of the numbers 6, 9, 15 they become proportional

Unit One Activity Proportion in our life

► **Activity Goal:** Possessing the skill of employing the concept of proportion in life situations.

► Implementation Steps:

Divide the class into groups, each group consisting of 3 students, ask them to implement the following steps:

- Each group uses the opposite table, suggests names for some items, and thus determines the situation expressed by the data of this table.
- Search for the unknown values in the table, then find the appropriate and correct values so that the quantities are proportional for the different items.
- If there is a fourth item and the quantity is 15 and the price is 70, is this item considered proportional to the first three items?
- Clarify the steps for answering the previous questions with an explanation of the validity of the results reached.



	Quantity / Number	Price / Cost
First Item	4	40
Second Item		60
Third Item	5	

UNIT 2

Algebra

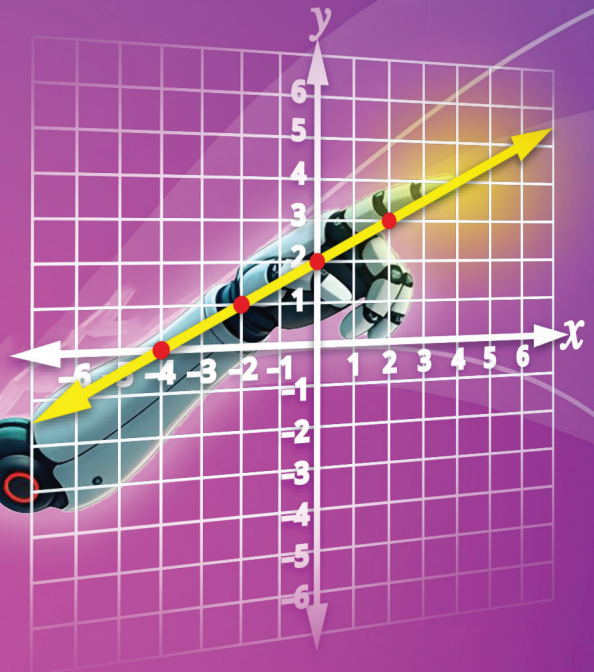
Unit Lessons

2-1 Cartesian Product

2-2 Relation and Function

2-3 Linear Function

2-4 Equations and Inequalities of the First Degree in One Variable



The concept of linear function is considered among the mathematical foundations upon which the study of smart models is based, which appear clearly in Artificial Intelligence applications, as smart models depend on predicting results based on mathematical relations between variables.

► Can the concepts and laws of relations and linear functions be used in artificial intelligence applications?

Issues and Life Skills

- Mathematical Communication
- Critical Thinking
- Sustainable Development
- Creative Thinking

Values

- Art Appreciation
- Civilizational Awareness
- Saving
- Respecting Others
- Teamwork

Lesson 2 - 1

Cartesian Product

■ Learning Outcomes

- Understand the concept of cartesian product.
- Understand the concept of ordered pair.
- Know the condition of equality of two ordered pairs.
- Represent the cartesian product of two sets by the arrow diagram (mapping) and the cartesian diagram.
- Use the cartesian product to solve life problems.

■ Vocabulary

- Ordered pair
- First projection
- Second projection
- Cartesian product
- Arrow diagram
- Cartesian diagram

■ Critical Thinking

Mention the difference between: The ordered pair (a, b) and the set $\{a, b\}$.

Get Ready!



Chess is considered one of the most famous mental games that require high concentration, logical thinking, and accurate planning, and it works on developing the skills of analysis, decision making, and problem-solving.

It is played on a board consisting of eight rows numbered from 1 to 8, and eight columns bearing letters from A to H, and each row intersects with each column in one square out of 64 squares on the board.

How can you name each square to determine its location accurately on the board?



In this lesson, you will learn the concept of ordered pair, and cartesian product and how to represent it; which will enable you to solve such life problems.

Think & Discuss!



A store displays a phone in three colors: blue, gold, black, and with three storage capacities 128, 256, 512 gigabytes.

How many options are available to buy a phone of this type?



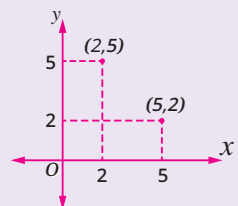
Learn!



The ordered pair

- The ordered pair (a, b) consists of two projections: The first projection a and the second projection b . The ordered pair expresses the position of a point in the orthogonal coordinate plane
- If $a \neq b$, then $(a, b) \neq (b, a)$
For example $(5, 2) \neq (2, 5)$

Because each of them expresses a different point in the coordinate plane.



Equality of two ordered pairs

If $(x, y) = (a, b)$ then $x = a$, $y = b$.

Example 1 Find the value of each of x, y in each of the following:

① $(x, y - 2) = (3, 3)$

② $(x + 3, xy) = (5, -8)$

③ $(3x^2, \frac{y^3}{2}) = (12, -32)$

④ $(\sqrt{x}, \frac{y^2}{5}) = (3, 4)$

■ Self-Evaluation ①

Find the values of the variables in each of the following:

- ① $(2x, -3y) = (8, 6)$
- ② $(2a + 1, 8) = (7, \frac{1}{2}b)$
- ③ $(x^2 + 1, \sqrt[3]{y}) = (5, -2)$

■ History of Mathematics

The cartesian product was named after the French scientist René Descartes, the founder of Analytic Geometry, which linked Algebra and Geometry using coordinates.



René Descartes
(1596 - 1650)

■ Self-Evaluation ②

If $X = \{1, 2, 3\}$,
 $Y = \{2, 5\}$

- ① Find : $X \times Y$ and represent it using an arrow diagram.
- ② Find : $Y \times X$ and represent it using a cartesian diagram.

■ Think with your classmates

If X, Y are two non-empty sets, is $X \times Y = Y \times X$?
If the answer is: No, when is that satisfied? Discuss.



$$\begin{aligned} \textcircled{1} \therefore (X, y - 2) &= (3, 3) \\ \therefore X &= 3 \\ y - 2 &= 3 \therefore y = 3 + 2 = 5 \end{aligned}$$

$$\begin{aligned} \textcircled{3} \therefore (3x^2, \frac{y^3}{2}) &= (12, -32) \\ \therefore 3x^2 &= 12 \therefore x^2 = 4 \\ \therefore x &= \pm\sqrt{4} = \pm 2 \\ \frac{y^3}{2} &= -32 \therefore y^3 = -64 \\ \therefore y &= \sqrt[3]{-64} = -4 \end{aligned}$$

$$\begin{aligned} \textcircled{2} \therefore (X + 3, xy) &= (5, -8) \\ \therefore X + 3 &= 5 \therefore X = 5 - 3 = 2 \\ xy &= -8 \therefore 2y = -8 \therefore y = -4 \end{aligned}$$

$$\begin{aligned} \textcircled{4} \therefore (\sqrt{x}, \frac{y^2}{5}) &= (3, 4) \\ \therefore \sqrt{x} &= 3 \therefore x = 9 \\ \frac{y^2}{5} &= 4 \therefore y^2 = 20 \\ \therefore y &= \pm\sqrt{20} = \pm 2\sqrt{5} \end{aligned}$$

Cartesian product of two finite sets

If X, Y are two non-empty sets, then:

- ① The cartesian product of set X by set Y is the set of all ordered pairs whose first projection is an element of X and whose second projection is an element of Y .
And it is denoted by $X \times Y$ and read as « X cross Y »
So that : $X \times Y = \{(x, y) : x \in X, y \in Y\}$
- ② The cartesian product of set Y by set X is the set of all ordered pairs whose first projection is an element of Y and whose second projection is an element of X .
And it is denoted by $Y \times X$ and read as « Y cross X »
So that : $Y \times X = \{(y, x) : y \in Y, x \in X\}$

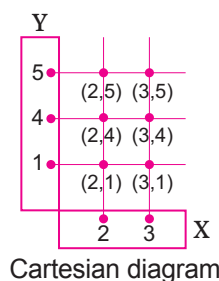
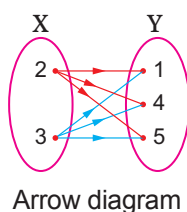
The cartesian product can be represented in two ways:
The arrow diagram and the cartesian diagram.

Example ②

If $X = \{2, 3\}$, $Y = \{1, 4, 5\}$ find $X \times Y$ and represent it using an arrow diagram and a cartesian diagram.



$$X \times Y = \{2, 3\} \times \{1, 4, 5\} = \{(2, 1), (2, 4), (2, 5), (3, 1), (3, 4), (3, 5)\}$$



► Try yourself to find $Y \times X$ and represent it using an arrow diagram and a cartesian diagram.

■ Critical Thinking

If X is a non-empty set, what is the cartesian product $X \times \emptyset$?

■ Self-Evaluation ③

① If $X = \{7, 8\}$

Find : X^2

And represent it using an arrow diagram.

② If $X = \{3, 4, 6\}$

Find : $X \times X$

And represent it using a cartesian diagram.

■ Self-Evaluation ④

If $X = \{5\}$, $Y = \{2, 3\}$,
 $Z = \{2, 4, 5\}$

Find each of the following:

① $Y \times (Y \cup Z)$

② $(X \cap Z) \times (Y \cap Z)$

③ $(Z - Y) \times (Z - X)$

Note

If X is a non-empty set, then : $X \times X = \{(x, y) : x \in X, y \in X\}$
And sometimes written as X^2 , and read as «X two».

Example ③

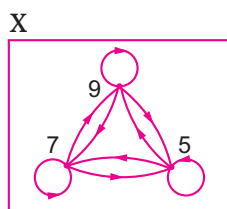
If $X = \{5, 7, 9\}$

Find X^2 and represent it using an arrow diagram and a cartesian diagram.

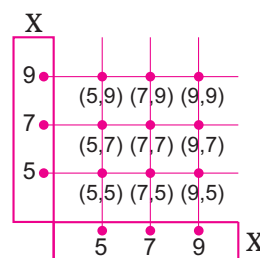


$$X^2 = X \times X = \{5, 7, 9\} \times \{5, 7, 9\}$$

$$= \{(5, 5), (5, 7), (5, 9), (7, 5), (7, 7), (7, 9), (9, 5), (9, 7), (9, 9)\}$$



Arrow diagram



Cartesian diagram

Example ④

If $X = \{-1, 0, 1\}$, $Y = \{0, 1, 2\}$, $Z = \{4\}$

Find each of the following :

① $(X \cup Y) \times Z$

② $(X \cap Y) \times Y$

③ $(X - Y) \times (Y - X)$



① $(X \cup Y) \times Z = \{-1, 0, 1, 2\} \times \{4\} = \{(-1, 4), (0, 4), (1, 4), (2, 4)\}$

② $(X \cap Y) \times Y = \{0, 1\} \times \{0, 1, 2\}$
 $= \{(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2)\}$

③ $(X - Y) \times (Y - X) = \{-1\} \times \{2\} = \{(-1, 2)\}$

Number of elements of cartesian product

If we denote the number of elements of set X by $n(X)$, and the number of elements of set Y by $n(Y)$, then:

$$n(X \times Y) = n(X) \times n(Y) \quad , \quad n(X^2) = (n(X))^2$$

For example:

If $X = \{0, 1, 3\}$, $Y = \{7, 8\}$.

Then $n(X) = 3$, $n(Y) = 2$

and $n(X \times Y) = n(Y \times X) = 3 \times 2 = 6$, $n(X^2) = 3^2 = 9$, $n(Y^2) = 2^2 = 4$

■ Self-Evaluation ⑤

If $X = \{1, 2, 3\}$,

$$n(X \times Y) = 21$$

Find: $n(Y^2)$

■ Think with your classmates

To which of the following cartesian products:

$$N \times N, Z \times Z, Q \times Q, R \times R$$

each of the following

ordered pairs belong to:

(Write all possible products)

① $(\sqrt{4}, 5)$

② $(\sqrt[3]{-8}, 3)$

③ $(2\frac{1}{2}, \frac{1}{3})$

④ $(\sqrt{5}, 1)$

Example 5

If $Z = \{1, 5\}$, $n(X \times Y) = 12$, $n(X^2) = 16$

Find: $n(Y \times Z)$



$$\therefore n(X^2) = 16$$

$$\therefore n(X) = 4$$

$$\therefore n(X \times Y) = n(X) \times n(Y) \quad \therefore 12 = 4 \times n(Y) \quad \therefore n(Y) = 3$$

$$\therefore n(Z) = 2$$

$$\therefore n(Y \times Z) = 3 \times 2 = 6$$

Cartesian product of two infinite sets

If X is an infinite set, then $X \times X$ is also an infinite set.

Examples of that are:

$N \times N = \{(x, y) : x \in N, y \in N\}$ where N is the set of natural numbers.

$Z \times Z = \{(x, y) : x \in Z, y \in Z\}$ where Z is the set of integers.

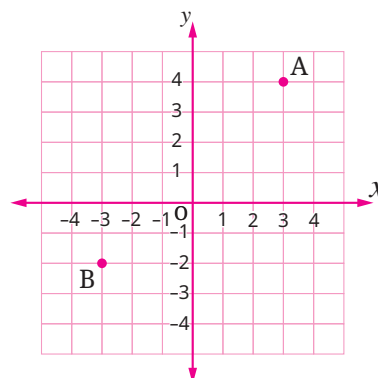
$Q \times Q = \{(x, y) : x \in Q, y \in Q\}$ where Q is the set of rational numbers.

$R \times R = \{(x, y) : x \in R, y \in R\}$ where R is the set of real numbers.

For Example:

► The opposite figure represents a small part of the orthogonal grid of the cartesian product $Z \times Z$ and every point of intersection points represents one of the ordered pairs in $Z \times Z$ where:

$$A(3, 4) \in Z \times Z, B(-3, -2) \in Z \times Z$$



Lesson Assessment



First

Measuring Conceptual Understanding



► Choose the correct answer from the given ones:

① If $(a, 3) = (-2, b)$, then $a + b = \dots\dots\dots$

(a) -5 (b) -1 (c) 1 (d) 5

③ $\{3\} \times \{5\} = \dots\dots\dots$

(a) $\{15\}$

(b) $\{3, 5\}$

(c) $\{3, 5\}$

(d) $\{(3, 5)\}$

② If $(X, y - 1) = (4, 3)$, then $\frac{x}{y} = \dots\dots\dots$

(a) 1 (b) 2 (c) 3 (d) 4

④ If $X = \{a, b, c\}$, $Y = \{2, 3, 5\}$

then $(a, 5) \dots\dots\dots Y \times X$

(a) $\in$ (b) $\notin$ (c) $\subset$ (d) $\not\subset$

- ⑤ If $X = \{3, 5\}$, $Y = \{6, 7\}$,
then $(6, 7) \in \dots\dots\dots$
(a) $X \times Y$ (b) $Y \times X$
(c) X^2 (d) Y^2
- ⑦ If $X = \{5, 7, 9\}$ and $n(X \times Y) = 6$
then $n(Y^2) = \dots\dots\dots$
(a) 4 (b) 6 (c) 9 (d) 12
- ⑨ If $X = \{2, 3, 4, 7, 8\}$, $Y = \{1, 4, 5, 6\}$,
then $n(X^2) - n(Y \times X) = \dots\dots\dots$
(a) 5 (b) 10 (c) 15 (d) 25
- ⑪ If $X \times Y = \{(a, 5), (4, 7), (9, b), (9, 5)\}$,
then $X = \dots\dots\dots$
(a) $\{7, 9\}$ (b) $\{4, 7\}$
(c) $\{4, 9\}$ (d) $\{5, 9\}$
- ⑥ To which cartesian product from the
following, the point $(\frac{1}{2}, \sqrt{5})$ belongs to?
(a) $N \times N$ (b) $Z \times Z$
(c) $Q \times Q$ (d) $R \times R$
- ⑧ If $X \times Y = \{(1, 4), (1, 7), (1, 8)\}$
then $n(Y^2) = \dots\dots\dots$
(a) 1 (b) 3 (c) 6 (d) 9
- ⑩ If $n(X^2) = 9$, $n(X \times Y) = 12$,
then $n(Y^2) = \dots\dots\dots$
(a) 4 (b) 6 (c) 9 (d) 16
- ⑫ If $X^2 = \{(4, 9), (a, a), (b, a), (b, b)\}$,
then $X = \dots\dots\dots$
(a) $\{2, 3\}$ (b) $\{4, 9\}$
(c) $\{4\}$ (d) $\{9\}$

Second

Applying Concepts



- ⑬ Find the value of each of X , Y in each of the following:
- ① $(X^2, 7) = (4, Y^2 - 2)$ ② $(5^{X-2}, XY) = (1, 18)$
- ⑭ If $(2a - 3, 4) = (4a, b + 5)$ find the value of $|2a - b|$
- ⑮ If $X = \{3, 2\}$, $Y = \{2, 4\}$ find each of the following and represent it using an arrow diagram and a cartesian diagram:
- ① $X \times Y$ ② $Y \times X$ ③ X^2 ④ Y^2
- ⑯ If $X = \{1, 2, 3\}$, $Y = \{X, Y\}$ find each of the following and represent it using an arrow diagram and a cartesian diagram:
- ① $X \times Y$ ② $Y \times X$ ③ X^2 ④ Y^2
- ⑰ If $X = \{6, 4, 8\}$, $Y = \{3, 4, 5\}$ find each of the following and represent it using an arrow diagram and a cartesian diagram:
- ① $(X \cap Y) \times Y$ ② $(X - Y) \times Y$ ③ $(Y - X) \times X$

18 If $X \times Y = \{(3, 3), (4, 3), (3, 7), (4, 7), (3, 8), (4, 8)\}$ find each of the following:

① X, Y

② $Y \times (X \cap Y)$

③ $(X - Y) \times (Y - X)$

④ $(X \cup Y) \times (X \cap Y)$

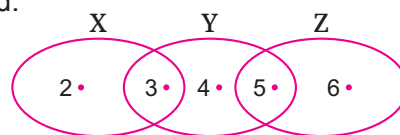
⑤ $n(Y^2)$

⑥ $n(X^2)$

19 The given figure represents Venn diagram for sets X, Y, Z find:

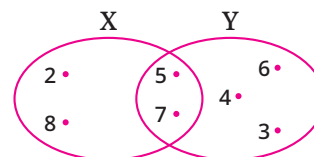
① $X \times (Y \cap Z)$

② $(X \cup Y) \times (Y - Z)$



20 The given figure represents Venn diagram for sets X, Y

find: $(X - Y) \times (Y - X)$



Third

Analysis and Subjects Integration



21 To number one of the cars belonging to a company, One number is chosen from the numbers $\{1, 3, 5, 7\}$ with one letter from the letters $\{a, b, c\}$, In how many ways can this car be numbered?



Traffic rules and instructions must be adhered to when driving the car to ensure the safety of individuals.

22 Probability:

If $X = \{1, 2, 3, \dots, 10\}$, an ordered pair was chosen randomly from the cartesian product $X \times X$, what is the probability that the chosen ordered pair belongs to the cartesian product $\{1, 2, 3\} \times \{1, 2, 3, 4\}$?

Creative Thinking



23 If $X = \{1, 2, 3\}$, $Y = \{a, b, c\}$

How many ordered pairs $(X, Y) \in X \times Y$ satisfying that the first projection is an odd number, and the second projection is not b ?

24 Choose the correct answer from the given ones:

If $n(X) = k$, $n(X \times Y) = k + 4$ then: $n(Y)$ cannot equal:

(a) 2

(b) 3

(c) 4

(d) 5

Evaluate your understanding!

How well do you understand Cartesian Product? Tick the right box


☐
☐
☐
☐

Lesson 2 - 2

Relation and Function

■ Learning Outcomes

- Understand the concept of relation.
- Express the relation in different ways.
- Understand the concept of relation domain, and range.
- Understand the concept of function.
- Determine when the relation is a function.
- Understand the concept of function domain, codomain, and range.
- Represent the function in different ways.
- Express the function in a symbolic way.

■ Vocabulary

- Relation
- Function
- Arrow diagram
- Graph
- Relation table
- Domain
- Range
- Codomain

Get Ready!



The Arab world includes 22 countries, and they are all included in the League of Arab States. Assuming X is the set of Arab countries, Y is the set of capitals of Arab countries, can you find the set of all ordered pairs whose first projection belongs to X and second projection belongs to Y , such that the first projection is the country name and the second projection is its capital name?

- Is this set the cartesian product $X \times Y$ or a subset of it?
- Can one of the Arab countries have two capitals? And can one of them not have a capital?



Emblem of the League of Arab States

In this lesson, you will learn the concepts of relation and function, and how to express them in different ways, and we will determine whether the relation is a function, which will enable you to solve such problems.

Think & Discuss!



If $X = \{1, 2, 3\}$, $Y = \{1, 2, 4, 6\}$

Find the cartesian product $X \times Y$

can you determine the ordered pairs (X, Y) that belong to the cartesian product $X \times Y$ and satisfy that $Y = 2X$? Discuss.

Learn!



First The relation

If X, Y are two non-empty sets, then the relation from set X to set Y is a subset of the cartesian product $X \times Y$ and we will denote it by the symbol R , where $R \subset X \times Y$ and it means a connection linking some or all elements of X with some or all elements of Y .

That means the relation from set X to set Y is a set of ordered pairs whose first projection belongs to X and whose second projection belongs to Y .

- The domain of the relation is the set of all first projections of the ordered pairs in the relation.

which means that: the domain of R is $\{X : (X, Y) \in R\}$

- The range of the relation is the set of all second projections of the ordered pairs in the relation.

which means that: the range of R is $\{Y : (X, Y) \in R\}$

■ Notation in Mathematics

For each $(x, y) \in R$ where $x \in X, y \in Y$ we express that as follows: $x R y$

■ Self-Evaluation ①

If $X = \{2, 4, 6, 8\}$, $Y = \{2, 3, 5, 7\}$ and R is a relation from X to Y where " $x R y$ " means " $x \geq y$ " for every $x \in X, y \in Y$

- ① Write the relation as a set of ordered pairs.
- ② State the domain and the range of the relation.
- ③ Represent the relation using an arrow diagram.
- ④ Represent the relation graphically.

■ Note that

If the relation R is from X to X , then we say that R is a relation on X and therefore $R \subset X \times X$

■ Self-Evaluation ②

If $X = \{1, 2, 5, 8\}$ and R is a relation on X where " $x R y$ " means " $x - y$ is a positive even number" for every $x \in X, y \in X$

- ① Write the relation R as a set of ordered pairs.
- ② State the domain and the range of the relation.
- ③ Represent the relation using an arrow diagram.
- ④ Represent the relation graphically.

Ways to express the relation

The relation can be expressed in one of the following ways:

- ① A set of ordered pairs.
- ② Relation table.
- ③ Arrow diagram of the relation.
- ④ Graphical representation of the relation.

The following example clarifies that.

Example 1

If $X = \{2, 4, 6\}$, $Y = \{1, 3, 5, 7\}$ and R is a relation from X to Y where " $x R y$ " means " $x + y = 7$ " for every $x \in X, y \in Y$

- ① Write the relation as a set of ordered pairs.
- ② Write the relation in a table form.
- ③ Represent the relation using an arrow diagram (mapping).
- ④ Represent the relation graphically.
- ⑤ State the domain and the range of the relation.

✓

① $R = \{(2, 5), (4, 3), (6, 1)\}$

③

②

x	2	4	6
y	5	3	1

④

⑤ The domain is $\{2, 4, 6\}$, the range is $\{1, 3, 5\}$

Example 2

If $X = \{2, 3, 5, 7\}$ and R is a relation on X where " $x R y$ " means " $y = x - 2$ " for every $x \in X, y \in X$

- ① Write R as a set of ordered pairs.
- ② State the domain and the range of the relation.
- ③ Represent the relation with an arrow diagram (mapping).
- ④ Represent the relation graphically.

✓

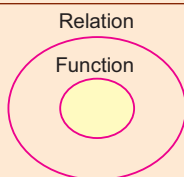
① $R = \{(5, 3), (7, 5)\}$

③

② The domain is $\{5, 7\}$, the range is $\{3, 5\}$

④

■ When does the relation become a function?



Not every relation is a function, but all functions are relations satisfying that:

- Every element of X appears as a first projection only once in the set of ordered pairs representing the relation or in the relation table.
- Every element of X has only one arrow coming out of it in the arrow diagram representing the relation.
- Every vertical line for every element of X passes through only one point in the graphical representation.

■ Self-Evaluation ③

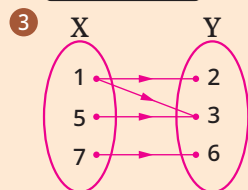
If $X = \{1, 5, 7\}$, $Y = \{2, 3, 6\}$ and each of the following relations is from X to Y

Determine which relation represents a function and which does not represent a function, stating the reason. And if it is a function, write its range.

① $R = \{(1, 3), (5, 6), (7, 2)\}$

②

x	5	7
y	3	6



Second The function

The function from set X to set Y is a relation from X to Y relates every element of X to only one element of Y and:

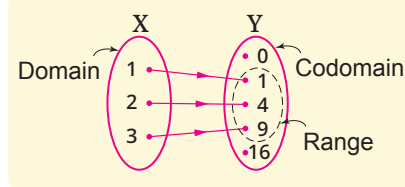
- Set X is called "**Function Domain**".
- Set Y is called "**Function Codomain**".
- The set of images of the elements of the domain X is called "**Function Range**", and it is a subset of the codomain Y .

For example:

If $X = \{1, 2, 3\}$, $Y = \{0, 1, 4, 9, 16\}$ and R is a relation from X to Y where " $x R y$ " means that " $y = x^2$ " for every $x \in X, y \in Y$

Then the relation R represents a "Function" because every element of X relates to only one element of Y and:

- The domain = $X = \{1, 2, 3\}$
- The codomain = $Y = \{0, 1, 4, 9, 16\}$
- The range = $\{1, 4, 9\}$



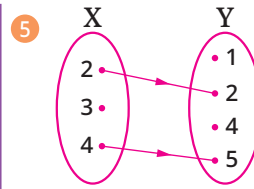
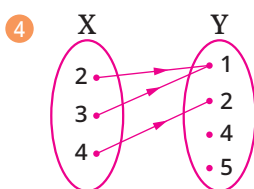
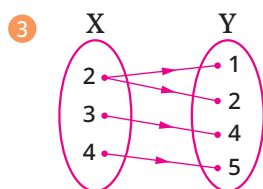
The arrow diagram

Example 3

If $X = \{2, 3, 4\}$, $Y = \{1, 2, 4, 5\}$ and each of the following relations is from X to Y . Determine which relation represents a function and which does not represent a function, stating the reason and if it is a function, write its range.

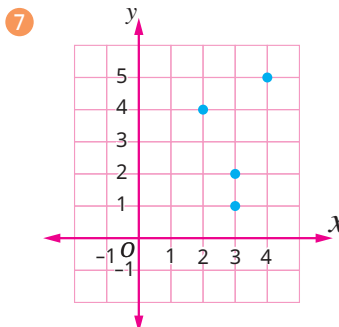
① $R = \{(2, 2), (3, 2), (4, 2)\}$

② $R = \{(2, 5), (3, 4)\}$



⑥

x	2	3	3	4
y	1	2	4	5



■ Self-Evaluation ④

If $X = \{3, 5, 7\}$,
 $Y = \{6, 9, 10, 16, 21\}$ and
 R is a relation from X to Y
 where " $X R Y$ " means
 $\frac{y}{x} = \text{a natural number}$
 for every $x \in X, y \in Y$

- ① Write R as a set of ordered pairs.
- ② Represent R using an arrow diagram.
- ③ Represent R graphically.
- ④ Does R represent a function or not? why? If it is a function, find its domain and range.



- ① The relation represents a function, because every element of X appears as a first projection only once, the range = $\{2\}$
- ② The relation does not represent a function, because element $4 \in X$ did not appear as a first projection.
- ③ The relation does not represent a function, because element $2 \in X$ had more than one arrow coming out of it.
- ④ The relation represents a function, because every element of X had only one arrow coming out of it, the range = $\{1, 2\}$
- ⑤ The relation does not represent a function because element $3 \in X$ did not have an arrow coming out of it.
- ⑥ The relation does not represent a function, because element $3 \in X$ appeared as a first projection more than once.
- ⑦ The relation does not represent a function, because the vertical line at $3 \in X$ passes through more than one point of the points representing the relation.

Example 4

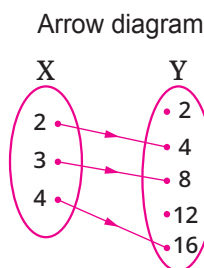
If $X = \{2, 3, 4\}$, $Y = \{2, 4, 8, 12, 16\}$ and R is a relation from X to Y where " $X R Y$ " means " $y = 2^x$ " for every $x \in X, y \in Y$

- ① Write R as a set of ordered pairs.
- ② Represent R using an arrow diagram (mapping).
- ③ Represent R graphically.
- ④ Does R represent a function or not? why? If it is a function find its domain and range.

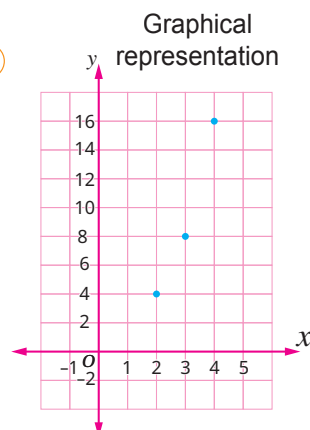


- ① $\because 2^2 = 4 \in Y \quad \therefore (2, 4) \in R$
 $2^3 = 8 \in Y \quad \therefore (3, 8) \in R$
 $2^4 = 16 \in Y \quad \therefore (4, 16) \in R$
 $\therefore R = \{(2, 4), (3, 8), (4, 16)\}$

②



③



- ④ R represents a function from X to Y because every element of X appears as a first projection only once in the ordered pairs representing the relation.

The Domain = $\{2, 3, 4\}$, The Range = $\{4, 8, 16\}$

■ Self-Evaluation ⑤

If $X = \{-2, -1, 0, 1, 2\}$ and R is a relation on X where " $X R Y$ " means " $X = -Y$ " for every $X \in X, Y \in X$

- ① Write R as a set of ordered pairs.
- ② Represent it using an arrow diagram (mapping).
- ③ Represent it graphically.
- ④ Does R represent a function or not? why?

■ Critical Thinking

In Example ⑤

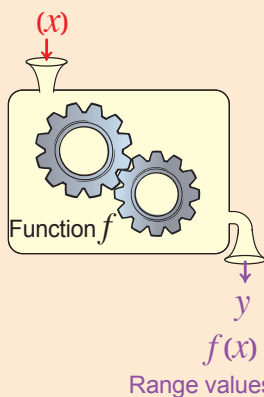
If $X = \{\frac{1}{2}, -1, 1, 2\}$

Does the relation on X represent a function or not? why?

■ Note

You can consider the function f as a machine into which we input the domain values (X) and it outputs the range values (Y) as follows:

Domain values



■ Note that

$f(X)$ is not f multiplied by (X) , it means the value of f at X , for example $f(2)$ means the value of f at $X = 2$

Example 5

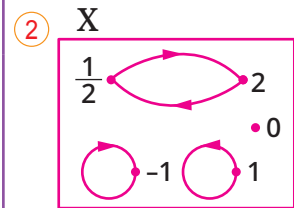
If $X = \{\frac{1}{2}, -1, 0, 1, 2\}$ and R is a relation on X where " $X R Y$ " means " $X Y = 1$ " for every $X \in X, Y \in X$

- ① Write R as a set of ordered pairs.
- ② Represent it using an arrow diagram.
- ③ Does R represent a function or not? why?



① $R = \{(\frac{1}{2}, 2), (-1, -1), (1, 1), (2, \frac{1}{2})\}$

③ R does not represent a function because element $0 \in X$ did not have an arrow coming out of it in the arrow diagram



Function notation

- The function is denoted by one of the symbols: f or g or ...
- The function f from set X to set Y is written mathematically as:
 $f: X \longrightarrow Y$ and read " f from X to Y "
- If the ordered pair (X, Y) belongs to the function f , then the element Y is the image of the element X under the function f and we express that by "The function rule" or "The equation" : $y = f(x)$
- X is called the Independent variable, and Y is called the dependent variable.

For example:

If $X = \{2, 3, 4\}$, $Y = \{1, 4, 9, 16\}$ and f is a function from X to Y Where $f(X) = X^2$, then:

At $X = 2$:
 $f(2) = 2^2 = 4$

At $X = 3$:
 $f(3) = 3^2 = 9$

At $X = 4$:
 $f(4) = 4^2 = 16$

Therefore, the range of the function is $\{4, 9, 16\}$

Example 6

If $X = \{-2, -1, 0, 1, 2\}$, $Y = \{y : y \in Z, -1 \leq y \leq 7\}$ and the function $f: X \longrightarrow Y$ where $f(X) = 3 - 2X$

- ① Find the images of elements of X under the function f stating the range of the function.
- ② Write the function in a table form.
- ③ Represent the function graphically.

Self-Evaluation ⑥

If $X = \{1, 2, 3, 5\}$,
 $Y = \{3, 4, 5, 6, 7, 8, 10\}$
 and the function $f: X \rightarrow Y$
 where $f(x) = x + 3$

- ① Find the images of elements of X under the function f stating the range of the function.
- ② Write the function in a table form.
- ③ Represent the function graphically.

Note that

If the function domain is not mentioned, we consider the domain is the set of all real numbers that make the function defined.

Self-Evaluation ⑦

If $f(x) = -3x + 4$, find:

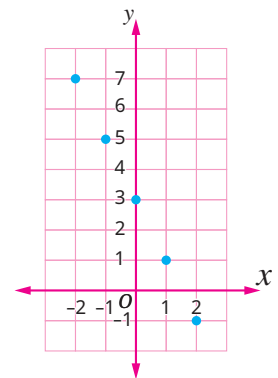
- ① $f(-2) + f(2)$
- ② $3f(5) + 5f(3)$



- ① At $X = -2$ then: $f(-2) = 3 - 2(-2) = 7$
 At $X = -1$ then: $f(-1) = 3 - 2(-1) = 5$
 At $X = 0$ then: $f(0) = 3 - 2(0) = 3$
 At $X = 1$ then: $f(1) = 3 - 2(1) = 1$
 At $X = 2$ then: $f(2) = 3 - 2(2) = -1$
 $\therefore$ The range of the function is $\{7, 5, 3, 1, -1\}$

②	x	-2	-1	0	1	2
	y	7	5	3	1	-1

③



Example 7

If the function $f(x) = x^2 - 3$ find the value of each of:

- ① $f(0) - f(\frac{1}{3})$
- ② $2f(\sqrt{5}) + 5f(\sqrt{2})$



- ① $f(0) - f(\frac{1}{3}) = [(0)^2 - 3] - [(\frac{1}{3})^2 - 3] = \frac{-1}{9}$
- ② $2f(\sqrt{5}) + 5f(\sqrt{2}) = 2[(\sqrt{5})^2 - 3] + 5[(\sqrt{2})^2 - 3] = -1$

Lesson Assessment



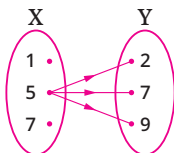
First

Measuring Conceptual Understanding

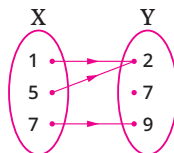


► Choose the correct answer from the given ones:

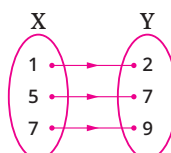
- ① Which of the following relations is not a function from X to Y ?



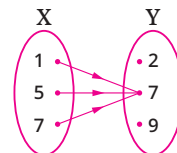
(a)



(b)

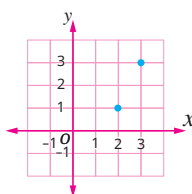


(c)

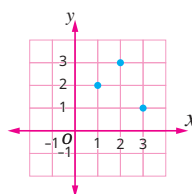


(d)

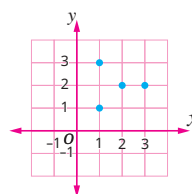
- ② If $X = \{1, 2, 3\}$, then which of the following relations represents a function on X ?



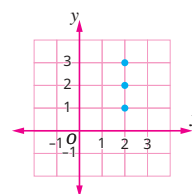
(a)



(b)



(c)



(d)

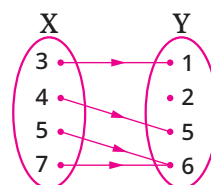
- ③ If $X = \{1, 2, 5\}$, $Y = \{6, 7, 9\}$, then which of the following relations represents a function from X to Y ?
- (a) $\{(1, 6), (2, 7), (2, 9), (5, 9)\}$ (b) $\{(9, 1), (6, 2), (7, 5)\}$
 (c) $\{(1, 7), (5, 9)\}$ (d) $\{(1, 7), (2, 7), (5, 7)\}$
- ④ If R is a relation where $R = \{(1, 7), (2, 8), (3, 7), (5, 8)\}$ what is the range of the relation?
- (a) $\{1, 2, 3, 5\}$ (b) $\{7, 8, 5\}$ (c) $\{7, 8\}$ (d) $\{1, 2, 7, 8\}$
- ⑤ What is the domain of the function $\{(1, 2), (2, 3), (4, 2), (5, 3)\}$?
- (a) $\{2, 3\}$ (b) $\{1, 2, 3, 4\}$ (c) $\{1, 2, 4, 5\}$ (d) $\{2, 3, 4, 5\}$
- ⑥ If $X = \{5, 7, 8\}$, $Y = \{9, 10\}$ and R is a function from X to Y where $R = \{(5, 9), (8, 9), (k, 9)\}$, what is the value of k ?
- (a) 5 (b) 7 (c) 8 (d) 9
- ⑦ If $X = \{2, 3, 4\}$ and R is a function on X where $R = \{(a, 4), (b, 2), (3, 4)\}$ what is the value of $a + b$?
- (a) 4 (b) 5 (c) 6 (d) 7
- ⑧ If $f(x) = x^2$, $g(x) = x + 4$, what is the value of $5f(2) + 2g(5)$?
- (a) 18 (b) 20 (c) 38 (d) 70

Second

Applying Concepts

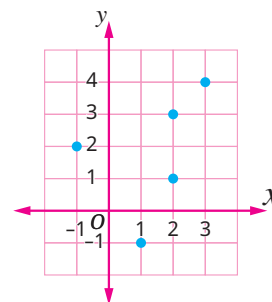


- ⑨ The given arrow diagram represents the relation R from X to Y .



- ① Write the relation as a set of ordered pairs.
- ② Write the relation in a table form.
- ③ Represent the relation graphically.
- ④ Find the domain and the range of the relation.
- ⑤ Does the relation represent a function or not? why?

- ⑩ If the given figure is the graphical representation for the relation R from X to Y where $X = \{-1, 1, 2, 3\}$, $Y = \{-1, 1, 2, 3, 4\}$



- ① Write the relation as a set of ordered pairs.
- ② Write the relation in a table form.
- ③ Represent the relation with an arrow diagram.
- ④ Find the domain and the range of the relation.
- ⑤ Does the relation represent a function or not? why?

- ⑪ If R is a relation from X to Y and $R = \{(2, 2), (0, 2), (3, 1), (4, 2)\}$

- ① Write the relation in a table form.
- ② Represent the relation graphically.
- ③ Find the domain and the range of the relation.

- ⑫ If R is a relation from X to Y where $X = \{1, 2, 3, 4\}$, $Y = \{3, 5, 7\}$ and " xRy " means " $(x + y)$ is a prime number" for every $x \in X, y \in Y$

- ① Write R as a set of ordered pairs.
- ② Write the relation in a table form.
- ③ Represent the relation using an arrow diagram.
- ④ Find the domain and the range of the relation.
- ⑤ Does the relation represent a function or not? why?

- ⑬ If R is a relation from X to Y where $X = \{2, 3, 4, 5\}$, $Y = \{3, 5, 6, 7, 9, 11\}$ and " $x R y$ " means " $y = 2x + 1$ " for every $x \in X, y \in Y$
- ① Write R as a set of ordered pairs.
 - ② Represent the relation using an arrow diagram.
 - ③ Represent the relation graphically.
 - ④ Does the relation represent a function or not? Why? If it is a function find its domain and its range.
- ⑭ If R is a relation on X where $X = \{1, 2, 3, 4, 5\}$ and " $x R y$ " means " $x + y = 5$ " for every $x \in X, y \in X$
- ① Write R as a set of ordered pairs.
 - ② Represent the relation using an arrow diagram.
 - ③ Find the domain and the range of the relation.
 - ④ Does the relation represent a function or not? Why?
- ⑮ If R is a relation on N where N is the set of natural numbers and " $x R y$ " means " $xy = 18$ " for every $x \in N, y \in N$
- ① Write R as a set of ordered pairs.
 - ② Write the relation in a table form.
 - ③ Write the domain and the range of the relation.
 - ④ If $m R 9$, find the value of m .
 - ⑤ If $6 R k$, find the value of k .
- ⑯ If $X = \{4, 6, 8\}$, $Y = \{-2, -1, 0, 1, 2\}$ and f is a function from X to Y where $f(x) = 4 - \frac{1}{2}x$. Write f as a set of ordered pairs, then find the range of the function and represent it graphically.
- ⑰ If $X = \{1, 2, 3, 4, 5, 6\}$, N is the set of natural numbers, and f is a function from X to N where $f(x) = x^2 - 1$, Write f as a set of ordered pairs, then find the range of the function.

Third

Analysis and Subjects Integration



- ⑱ Determine which of the following relations is considered a function and which is not:
- ① The relation between "student seat number" and "his exam grade".
 - ② The relation between "Sharm El-Sheikh hotels" and "tourists".
 - ③ The relation between "names of governorates" and "the administrative capital of each governorate".
 - ④ The relation between "students" and "school activities".

Creative Thinking



- ⑲ If R is a relation on X where $X = \{x : x \in N, 10 \leq x < 100\}$
Where " $x R y$ " means " $y = 5x$ " for every $x \in X, y \in X$
Find the number of elements of the relation R

Evaluate your understanding!

How well do you understand Relation and Function? Tick the right box


☐
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Lesson 2 - 3

Linear Function

■ Learning outcomes

- Understand the concept of linear function.
- Represent the linear function graphically.
- Understand the concept of constant function.
- Represent the constant function graphically.
- Using technology to represent the linear function and the constant function.
- Use the linear function, and the constant function to solve some life problems.

■ Vocabulary

- Linear function
- Constant function

■ Note that

- In general, the domain of the linear function is $\mathbb{R}$ but in some life applications the domain is a subset of $\mathbb{R}$ according to the data of each problem.

Get Ready!



Salah Al-Din Al-Ayyubi Citadel is one of the most important landmarks of Islamic Cairo, and one of the most ancient military castles. It includes many Islamic archaeological landmarks, including the Mosque of Muhammad Ali Pasha. If there were 120 visitors in the Citadel at 10 AM, and as the day goes on, 40 new visitors enter every hour, so the relation between the total number of visitors $f(t)$ and the time in hours (t) is given by $f(t) = 120 + 40t$



Muhammad Ali Mosque at the Citadel of Saladin

Can you represent this relation graphically, and determine after how many hours the total number of visitors will be 360?

In this lesson, you will learn some different forms of the function, including the linear function and the constant function, and the method of representing them graphically, which will enable you to solve such problems.

Think & Discuss!



Can you determine which of the following ordered pairs satisfies the function f where $f(x) = 4x - 5$?

(0, 1)

(2, 3)

(4, 10)

(5, 15)

Learn!



Linear Function

- The function f where $f(x) = ax + b$, a, b are real numbers, $a \neq 0$ is called a linear function.
- The linear function is a function of the first degree, and is represented graphically by a straight line cutting the y -axis at the point $(0, b)$, and cutting the x -axis at the point $(-\frac{b}{a}, 0)$.

Example:

The function f where $f(x) = x + 4$ is a linear function of the first degree, and is represented graphically by a straight line cutting the y -axis at the point $(0, 4)$, and cutting the x -axis at the point $(-4, 0)$.

■ Self-Evaluation ①

Represent graphically each of the following:

① $f(x) = 2x + 3$

② $g(x) = -\frac{1}{2}x$

■ Note that

When representing the linear function, it is sufficient to find two ordered pairs belong to the function, and you can find a third ordered pair to verify that the three points lie on one straight line.

■ Using Technology

- Several programs can be used to represent the linear function and study it, such as the GeoGebra program.
- Try yourself using GeoGebra program to represent the two functions in Example ①.



■ Note that

The function f where $f(x) = ax$, $a \neq 0$ represents graphically a straight line passing through the origin point $(0, 0)$.

Example ①

Represent graphically each of the following:

① $f(x) = 6 - 2x$

② $g(x) = \frac{1}{3}x$



- ① To represent the function f graphically, determine any three ordered pairs belong to the function f where $f(x) = 6 - 2x$, set any three values for (x) , let them be 0, 1, 2 and find the function at each value.

At : $x = 0$

$$\therefore f(0) = 6 - 2(0) = 6$$

$$\therefore (0, 6) \in f$$

At : $x = 1$

$$\therefore f(1) = 6 - 2(1) = 4$$

$$\therefore (1, 4) \in f$$

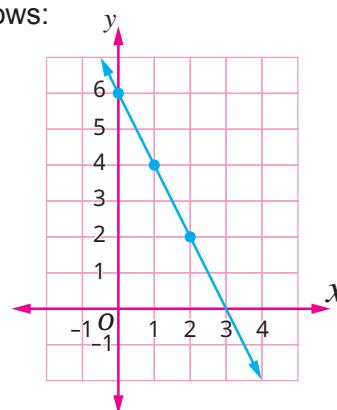
At : $x = 2$

$$\therefore f(2) = 6 - 2(2) = 2$$

$$\therefore (2, 2) \in f$$

Arrange the ordered pairs in a table as follows:

x	0	1	2
y	6	4	2



■ Note that

You can find the two points where the line intersects with the coordinate axes and use them in the graphical representation. For example, when representing $f(x) = 6 - 2x$:

- The point of intersection with y -axis is $(0, b)$ that equals $(0, 6)$.
- The point of intersection with x -axis is $(-\frac{b}{a}, 0)$ that equals $(3, 0)$.

■ Note that

If the coefficient of x is a fraction, it is preferable to choose numbers divisible by the denominator of this fraction to facilitate the representation.

- ② To determine three ordered pairs belong to the function g where $g(x) = \frac{1}{3}x$.

At : $x = 0$

$$\therefore g(0) = \frac{1}{3}(0) = 0$$

$$\therefore (0, 0) \in g$$

At : $x = 3$

$$\therefore g(3) = \frac{1}{3}(3) = 1$$

$$\therefore (3, 1) \in g$$

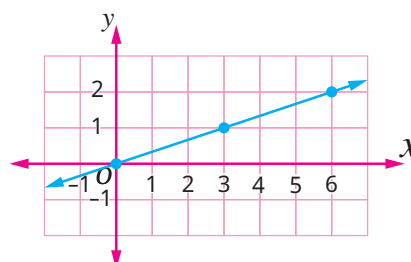
At : $x = 6$

$$\therefore g(6) = \frac{1}{3}(6) = 2$$

$$\therefore (6, 2) \in g$$

Arrange the ordered pairs in a table as follows:

x	0	3	6
y	0	1	2



■ Special Case

If $b = 0$, then the function f where $f(x) = 0$ is a constant function having no degree, and is represented graphically by the X -axis.

■ Self-Evaluation ②

Represent graphically each of the following:

① $f(x) = -1$

② $g(x) = 4$

■ Self-Evaluation ③

If $f(x) = ax + 3$ and the straight line representing the function f cuts the X -axis at the point $(\frac{3}{4}, 0)$, find the value of each of a , $f(-1)$.

Constant function

- The function f where $f(x) = b$, b is a real number, is called a constant function.
- If $b \neq 0$, then the constant function is of the zero degree, and is represented graphically by a straight line parallel to the X -axis and passing through the point $(0, b)$.

For example:

The function f where $f(x) = 5$ is a constant function of zero degree, and is represented graphically by a straight line parallel to the X -axis and passing through the point $(0, 5)$.

Example 2

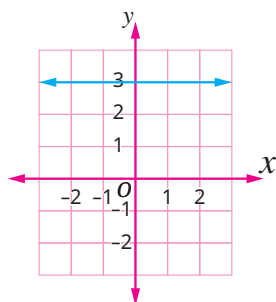
Represent graphically each of the following:

① $f(x) = 3$

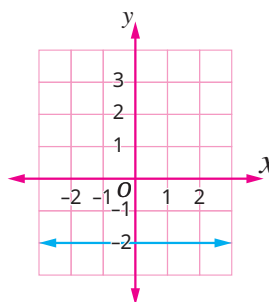
② $g(x) = -2$



① $f(x) = 3$



② $g(x) = -2$



Applications on Linear function

Example 3

If $f(x) = 3x + b$ and the point $(4, 7)$ lies on the straight line representing the function f . Find the value of each : b , $f(1)$



$$\begin{aligned} \because \text{The point } (4, 7) \text{ lies on the straight line representing the function } f \\ \therefore f(4) = 7 & \qquad \therefore 7 = 3 \times 4 + b & \qquad \therefore b = -5 \\ \therefore f(x) = 3x - 5 & \qquad \therefore f(1) = 3(1) - 5 = -2 \end{aligned}$$

Example 4

Industry : If the cost of producing a number of shirts is given by the function f where $f(x) = 50x + 100$. Where x is the number of shirts, $f(x)$ is the cost in pounds. Make a table showing some values of x and the corresponding cost and represent it graphically, then find the number of shirts that cost 700 pounds.

■ Self-Evaluation ④

If the cost of producing a number of books is given by the relation $f(x) = 10 + 5x$ where x is the number of books, $f(x)$ is the cost in pounds. Make a table showing some values of x and the corresponding cost, then represent it graphically, then find the number of books that can be produced at a cost of 90 pounds.

■ Self-Evaluation ⑤

A tank with a capacity of 216 liters is completely full of water, water leaks from it such that the relation between the amount of remaining water in the tank $f(t)$ in liters and the elapsed time (t) in hours is given by the function f where $f(t) = 216 - 6t$

- ① Represent this function graphically, stating the domain.
- ② When does the tank become completely empty?

✓ $\therefore f(x) = 50x + 100$

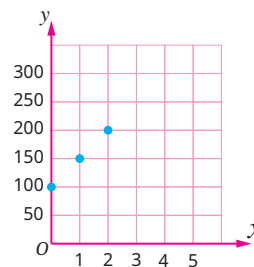
$\therefore f(0) = 100, f(1) = 150, f(2) = 200$

x	0	1	2
y	100	150	200

If the cost = 700 pounds, then: $f(x) = 700$

$\therefore 50x + 100 = 700 \quad \therefore x = 12$

Meaning that the number of shirts is 12 shirts.



■ Note that

At $x = 0$, $y = 100$ which means that when the number of produced shirts equals zero, the cost equals 100 pounds, and this cost is fixed and it does not depend on the number of shirts.

■ Note in the previous example

The values of x cannot be negative or non-integer positive numbers, as it is not possible to produce (-3) shirts or (1.5) shirts, and therefore the domain is the set of natural numbers N , and the function is represented by discrete points. The domain may have a maximum limit when the daily production of the factory cannot exceed 100 shirts for example, so the domain is $\{0, 1, 2, 3, \dots, 100\}$

Example 5

A car moves at a constant speed from a point 50 km away from Cairo. If the distance (d) in kilometers that the car is away from Cairo after a time (t) in hours is determined by the relation $d(t) = 100t + 50$. Represent the function d graphically, and find the time required for the car's distance from Cairo to be 550 km.



$\therefore d(t) = 100t + 50$

$\therefore d(0) = 50, d(1) = 150, d(2) = 250$

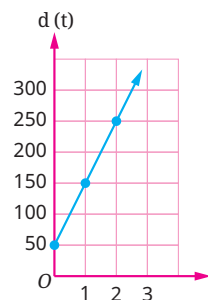
t	0	1	2
$d(t)$	50	150	250

If the distance from Cairo is 550 km:

$\therefore d(t) = 550$

$\therefore 100t + 50 = 550 \quad \therefore t = 5$

So the time is 5 hours.



■ Note in the previous example

The value of t cannot be negative and therefore the domain is $[0, \infty[$ and we represent the function by a ray whose start is the point $(0, 50)$.

Collaborative Activity



Use different sources (such as: the Internet, artificial intelligence applications, your school library or ...) and cooperate with your classmates in searching for relations in different sciences, each representing a linear function, and use one of the computer programs such as GeoGebra to represent each of them.

Lesson Assessment



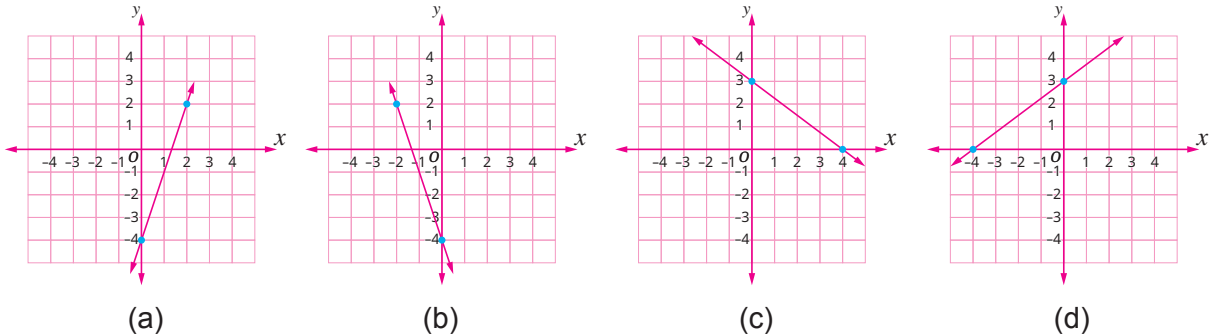
First

Measuring Conceptual Understanding



► Choose the correct answer from the given ones:

- ① Which of the following figures is considered a graphical representation of the function f where $f(x) = 3x - 4$?



- ② The straight line representing the function f where $f(x) = 3x - 12$ cuts the x -axis at the point:
 (a) $(-12, 0)$ (b) $(12, 0)$ (c) $(4, 0)$ (d) $(-4, 0)$
- ③ The straight line representing the function f where $f(x) = 6 - 2x$ cuts the y -axis at the point:
 (a) $(0, -6)$ (b) $(0, -3)$ (c) $(0, 3)$ (d) $(0, 6)$
- ④ The ordered pair $(1, 2)$ belongs to the function f where:
 (a) $f(x) = 2x - 1$ (b) $f(x) = -3x + 7$ (c) $f(x) = 2x + 1$ (d) $f(x) = -3x + 5$
- ⑤ If $f: R \longrightarrow R$ and $f(x) = 7$, what is the value of $\frac{5f(4)}{10f(6)}$?
 (a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{7}{3}$ (d) $\frac{7}{2}$

Second

Applying Concepts



► Choose the correct answer from the given ones:

- ⑥ If the straight line representing the function f where $f(x) = kx + 4$ passes through the point $(1, 1)$, what is the value of $f(-1)$?
 (a) -7 (b) -1 (c) 1 (d) 7
- ⑦ If the straight line representing the function f where $f(x) = 2x + b$ passes through the point $(-1, 5)$, what is the value of $f(0)$?
 (a) -7 (b) -5 (c) 5 (d) 7
- ⑧ If the point $(k, 5)$ lies on the straight line representing the function f where $f(x) = 2x - 7$ what is the value of k ?
 (a) 3 (b) 4 (c) 5 (d) 6
- ⑨ If $(5k, k)$ belongs to the function f where $f(x) = 3x + 14$, what is the value of k ?
 (a) 14 (b) 10 (c) -7 (d) -1

► Answer the following questions:

⑩ Represent graphically each of the following:

① $f(x) = 3x$

② $f(x) = \frac{1}{2}x + 3$

③ $f(x) = 2 - x$

④ $f(x) = 2$

⑤ $f(x) = 2x - 5$

⑥ $f(x) = 5 - 3x$

⑦ $f(x) = -\frac{1}{3}x + 2$

⑪ If $f(x) = 2x + 3$ and $f(k) = 11$, find the value of k

⑫ The given table represents a set of ordered pairs for the function f where $f(x) = 4x + 5$, find the value of $(a - b)$

x	3	a	8
$f(x)$	17	29	b

Third

Analysis and Subjects Integration



⑬ **Commerce:** The price of a photocopier after t years is given, in pounds, by the function f where $f(t) = 12000 - 750t$, find the price of the photocopier after 4 years of purchasing. Find after how many years the price of the photocopier becomes 3000 pounds.



Exercising regularly maintains health, develops the spirit of perseverance, and teaches discipline and teamwork.

⑭ **Sports:**

If the cost of renting a football field in pounds is given by the function f where $f(t) = 120t + 80$ where t is the number of hours of renting the field. Find the cost of renting the field for 2 hours. After how many hours will the cost be 680 pounds?

⑮ **Shopping:**

In supermarkets, you will often find shopping baskets stacked inside each other. If the baskets are all identical and their height are measured in centimeters while they are nested is given by the function f where $f(n) = 3n + 20$, where n is the number of nested baskets. Find the height of the baskets when there are 10 baskets. Then find the number of baskets if their height is 2 meters. Represent graphically the function f



Unconscious buying exhausts the individual and negatively affects the economy.

Creative Thinking



► Answer the following questions:

⑯ If $g(x) = 4$, $f(x) = -3$, what is the perpendicular distance between the two straight lines representing the two functions f, g ?

(a) 1

(b) 3

(c) 5

(d) 7

Evaluate your understanding!

How well do you understand the linear function?

Tick the right box


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Lesson 2 - 4

Equations and Inequalities of the First Degree in One Variable

■ Learning Outcomes

- Understand the concept of equation of the first degree in one variable.
- Solve the equation of the first degree in one variable in R .
- Understand the concept of inequality of the first degree in one variable.
- Solve the inequality of the first degree in one variable in R .
- Use equations and inequalities to solve some life problems.

■ Vocabulary

- Equation
- Inequality
- First degree

■ Note that

Equation of the first degree in one variable has a single variable raised to the power of one.

Get Ready!



Mohamed is keen on exercising regularly to maintain his good health and physical fitness. Mohamed wanted to join a sports club near his home. Subscription in this club requires paying 100 pounds fixed registration fees, in addition to 20 pounds for each training day.

Mohamed allocates 300 pounds monthly for exercising in the club, how many days can he train without exceeding this amount?



In this lesson, you will recognize how to solve equations and inequalities of the first degree in one variable, which will enable you to solve such life problems.

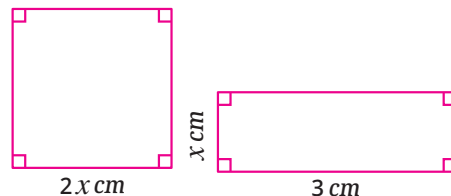
Think & Discuss!



In the opposite figure:

A square with side length $2x$ cm, and a rectangle with dimensions x cm, 3 cm.

What is the value of x that makes the perimeter of the square equal to the perimeter of the rectangle?



Learn!



First Equation of the first degree in one variable

The general form of the equation of the first degree in one variable is:

$$aX + b = 0$$

Where a, b are real numbers, $a \neq 0$

Examples:

Equations in one variable of the first degree

$$2X + 10 = 0$$

$$\frac{1}{2}n - 3 = 5$$

$$-4(8 + y) = 90$$

Equations in one variable not of the first degree

$$2\sqrt{x} - 0.5 = 0$$

$$x^2 - 1 = 3$$

$$\frac{1}{x} + 4 = x - 3$$

■ Self-Evaluation ①

Find the solution set of each of the following two equations in R :

① $4x - 5\sqrt{3} = 7\sqrt{3}$

② $3(\sqrt{5}x - 2) - 1 = 8$

■ Previous Knowledge

Properties of equality relation:

If a, b, c are three real numbers, $a = b$ then:

① $a + c = b + c$

② $a - c = b - c$

③ $ac = bc$

④ $\frac{a}{c} = \frac{b}{c}$ where $c \neq 0$

■ Critical Thinking

Find the solution set of each of the following two equations in R :

① $2(x + 1) + 1 = 2x + 3$

② $3x + 2 = 4(x + 1) - x$

■ Note that

Inequality of the first degree in one variable has a single variable raised to the power of one.

Solving equation of the first degree in one variable in R

- Solving an equation means finding the values of the variable that satisfy the equation.
- Equation of the first degree in one variable has a unique solution in R

Example 1

Find the solution set of each of the following two equations in R :

① $6x + 2 = 4x - 4$

② $5(\sqrt{3}x - 7) + 11 = 6$



① $\therefore 6x + 2 = 4x - 4$

$\therefore 6x - 4x = -4 - 2$

$\therefore 2x = -6$

$\therefore x = \frac{-6}{2}$

$\therefore x = -3$

$\therefore$ The solution set = $\{-3\}$

② $\therefore 5(\sqrt{3}x - 7) + 11 = 6$

$\therefore 5\sqrt{3}x - 35 + 11 = 6$

$\therefore 5\sqrt{3}x - 24 = 6$

$\therefore 5\sqrt{3}x = 6 + 24$

$\therefore 5\sqrt{3}x = 30 \quad \therefore x = \frac{30}{5\sqrt{3}}$

$\therefore x = \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$\therefore x = \frac{6\sqrt{3}}{3} \quad \therefore x = 2\sqrt{3}$

$\therefore$ The solution set = $\{2\sqrt{3}\}$

Second Inequality of the first degree in one variable

The general form of the Inequality of the first degree in one variable is:

$$ax + b < 0$$

Where a, b are real numbers, $a \neq 0$

Noting that the inequality sign can be one of the signs ($\leq, \geq, <, >$)

Examples:

Inequalities in one variable of the first degree

$$2x - 3 > 0$$

$$x + 4 < 2x$$

$$\frac{m+2}{2} \geq \frac{m+1}{4}$$

$$2(p-1) + 3 \leq 4p + 6$$

Inequalities in one variable not of the first degree

$$x^2 - 3x \geq -2$$

$$\frac{1}{y} - 4 < 5$$

$$2\sqrt{x} + 1 > 5$$

■ Self-Evaluation ②

Find the solution set of each of the following inequalities in R and represent it on the number line:

- ① $3x - 4 \geq 5$
- ② $6 - 3y \leq 2(y - 2)$
- ③ $\frac{2x}{3} - \frac{x}{6} < -2$

■ Previous Knowledge

Properties of inequality relation:

If a, b, c are three real numbers, $a < b$ then:

- ① $a + c < b + c$
- ② $a - c < b - c$
- ③ $ac < bc$ if : $c > 0$
 $ac > bc$ if : $c < 0$
- ④ $\frac{a}{c} < \frac{b}{c}$ if : $c > 0$
 $\frac{a}{c} > \frac{b}{c}$ if : $c < 0$

Solving inequality of the first degree in one variable in R

Solving an inequality means finding the values of the variable that satisfy the inequality.

Example 2

Find the solution set of each of the following inequalities in R and represent it on the number line :

- ① $5x + 2 > 12$
- ② $12 \geq 5x - 3$
- ③ $2x - 3 \leq 5x + 9$
- ④ $2(3x - 7) < 4x - 11$



$$\textcircled{1} \because 5x + 2 > 12 \quad \therefore 5x > 12 - 2 \quad \therefore 5x > 10$$

$$\therefore x > \frac{10}{5} \quad \therefore x > 2$$

$$\therefore \text{The solution set} =]2, \infty[$$



$$\textcircled{2} \because 5x - 3 \leq 12 \quad \therefore 5x \leq 12 + 3 \quad \therefore 5x \leq 15$$

$$\therefore x \leq \frac{15}{5} \quad \therefore x \leq 3$$

$$\therefore \text{The solution set} =]-\infty, 3]$$



$$\textcircled{3} \because 2x - 3 \leq 5x + 9 \quad \therefore 2x - 5x \leq 9 + 3 \quad \therefore -3x \leq 12$$

$$\therefore x \geq \frac{12}{-3} \quad \therefore x \geq -4$$

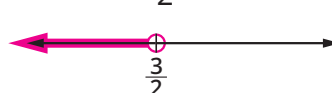
$$\therefore \text{The solution set} = [-4, \infty[$$



$$\textcircled{4} \because 2(3x - 7) < 4x - 11 \quad \therefore 6x - 14 < 4x - 11$$

$$\therefore 6x - 4x < -11 + 14 \quad \therefore 2x < 3 \quad \therefore x < \frac{3}{2}$$

$$\therefore \text{The solution set} =]-\infty, \frac{3}{2}[$$



Note:

The inequality $a < x < b$ is called a compound inequality, and it can be divided into the two inequalities $x > a$, $x < b$

For example:

The set of real numbers greater than 2 and less than 5 can be expressed in one of the following two ways:

$$\textcircled{1} x > 2, x < 5$$

$$\textcircled{2} 2 < x < 5$$



and it can be represented on the number line as shown in the figure.

■ Self-Evaluation ③

Use inequalities to express each of the following in two ways and represent it on the number line:

- ① The set of real numbers greater than 5 and less than 9
- ② The set of real numbers greater than -2 and less than or equal to 4.
- ③ The set of real numbers less than 8 and greater than or equal to -1.
- ④ The set of real numbers greater than or equal to -2 and less than or equal to 0.

■ Self-Evaluation ④

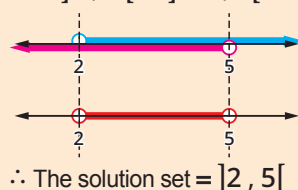
Find the solution set of each of the following inequalities in R and represent it on the number line:

- ① $4 \leq x + 2 < 8$
- ② $-7 < 5x + 3 < 18$
- ③ $1 \leq 4 - 3(x - 1) < 13$

■ Note that

When dividing the inequality into two inequalities as in ① the solution set is the intersection set of the two solution sets of these two inequalities, as follows:

$$\begin{aligned} \therefore x > 2, x < 5 \\ \therefore x \in]2, \infty[, x \in]-\infty, 5[\\ \therefore \text{The solution set} \\ =]2, \infty[\cap]-\infty, 5[\end{aligned}$$



■ Think with your classmates

Try with your classmates to solve the Inequality number ② by dividing it into two Inequalities.

Example 3

Use inequalities to express each of the following in two ways and represent it on the number line :

- ① The set of real numbers less than 7 and greater than or equal to 0
- ② The set of real numbers greater than or equal to 3 and less than 6
- ③ The set of real numbers less than or equal to -2 and greater than or equal to -6

	First method	Second method	Representation on the number line
①	$x < 7, x \geq 0$	$0 \leq x < 7$	
②	$x \geq 3, x < 6$	$3 \leq x < 6$	
③	$x \leq -2, x \geq -6$	$-6 \leq x \leq -2$	

Example 4

Find the solution set of each of the following inequalities in R , and represent it on the number line :

- ① $-1 < x - 3 < 2$
- ② $-8 < 3x - 2 \leq 7$
- ③ $-3 \leq 3(x - 7) - 3 \leq 6$

① $\therefore -1 < x - 3 < 2$
 $\therefore x - 3 > -1, x - 3 < 2$
 $\therefore x > -1 + 3, x < 2 + 3$
 $\therefore x > 2, x < 5$
 $\therefore 2 < x < 5$
 $\therefore \text{The solution set} =]2, 5[$

■ Diversified Strategies

The solution set can be found directly without dividing the inequality into two inequalities as follows applying the properties of inequality relation:

$$\begin{aligned} \therefore -1 < x - 3 < 2 \\ \therefore -1 + 3 < x < 2 + 3 \\ \therefore 2 < x < 5 \\ \therefore \text{The solution set} =]2, 5[\end{aligned}$$

② $\therefore -8 < 3x - 2 \leq 7$ $\therefore -8 + 2 < 3x \leq 7 + 2$
 $\therefore -6 < 3x \leq 9$ $\therefore \frac{-6}{3} < x \leq \frac{9}{3}$ $\therefore -2 < x \leq 3$
 $\therefore \text{The solution set} =]-2, 3]$

③ $\therefore -3 \leq 3(x - 7) - 3 \leq 6$ $\therefore -3 \leq 3x - 21 - 3 \leq 6$
 $\therefore -3 \leq 3x - 24 \leq 6$ $\therefore -3 + 24 \leq 3x \leq 6 + 24$
 $\therefore 21 \leq 3x \leq 30$ $\therefore \frac{21}{3} \leq x \leq \frac{30}{3}$ $\therefore 7 \leq x \leq 10$
 $\therefore \text{The solution set} = [7, 10]$

■ Self-Evaluation ⑤

Find the solution set of each of the following two inequalities in R and represent it on the number line:

①

$$x + 8 \geq 3x - 2 \geq 2 + x$$

②

$$5x + 17 < 4(5 + x) < 3x + 25$$

■ Critical Thinking

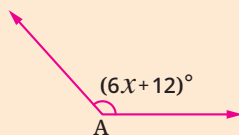
In Example ⑤

Can you solve the inequality ① by dividing it into two inequalities?

and can you solve the inequality ② by applying the properties of inequality relation without the need to divide the inequality into two inequalities?

■ Self-Evaluation ⑥

In the following figure:



$\angle A$ is an obtuse angle. What are the possible values of x ?

Example 5

Find the solution set of each of the following two inequalities in R and represent it on the number line:

① $4x - 1 < 3(x - 1) \leq 4x + 4$

② $2x - 19 \leq 3(x - 6) < x - 8$



① $\therefore 4x - 1 < 3(x - 1) \leq 4x + 4 \quad \therefore 4x - 1 < 3x - 3 \leq 4x + 4$
 $\therefore -1 < 3x - 3 - 4x \leq 4 \quad \therefore -1 < -x - 3 \leq 4$
 $\therefore -1 + 3 < -x \leq 4 + 3 \quad \therefore 2 < -x \leq 7 \quad \therefore -2 > x \geq -7$
 $\therefore$ The solution set = $[-7, -2[$



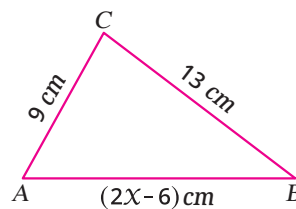
② $\therefore 2x - 19 \leq 3(x - 6) < x - 8 \quad \therefore 2x - 19 \leq 3x - 18 < x - 8$
 $\therefore 2x - 19 \leq 3x - 18, \quad 3x - 18 < x - 8$
 $\therefore 2x - 3x \leq 19 - 18, \quad 3x - x < 18 - 8$
 $\therefore -x \leq 1, \quad 2x < 10 \quad \therefore x \geq -1, \quad x < 5$
 $\therefore -1 \leq x < 5$
 $\therefore$ The solution set = $[-1, 5[$



Example 6

In the given figure:

What are the possible values of x ?



The possible values of x are the values that make the side lengths of the triangle satisfy the triangle inequality.

$$\therefore 13 - 9 < 2x - 6 < 13 + 9$$

$$\therefore 4 < 2x - 6 < 22$$

$$\therefore 4 + 6 < 2x < 22 + 6$$

$$\therefore 10 < 2x < 28$$

$$\therefore \frac{10}{2} < x < \frac{28}{2}$$

$$\therefore 5 < x < 14$$

$$\text{So that: } x \in]5, 14[$$

■ Remember that

The length of any side in a triangle is greater than the difference between the lengths of the other two sides and less than their sum.

Lesson Assessment



First

Measuring Conceptual Understanding



► Choose the correct answer from the given ones:

- ① Which of the following equations is an equation of the first degree in one variable?
 (a) $x + y = 12$ (b) $2x - \frac{2}{x} = 0$ (c) $2\sqrt{x} + 3 = 5$ (d) $2x - 4 = 3$
- ② Which of the following inequalities is an inequality of the first degree in one variable?
 (a) $x^2 - x \geq 3$ (b) $0.4t - 7 < 2t$ (c) $\frac{2}{t} + t > 5$ (d) $3x < 2y + 5$
- ③ What is the solution set of the equation $\sqrt{2}x - 1 = 1$ in R ?
 (a) $\{2\}$ (b) $\{1\}$ (c) $\{\sqrt{2}\}$ (d) $\emptyset$
- ④ Which of the following inequalities expresses the solution set in R represented by the given figure?

 (a) $x > 3$ (b) $x < 3$ (c) $x \geq 3$ (d) $x \leq 3$
- ⑤ What is the solution set of the inequality $2x < 14$ in R ?
 (a) $] -\infty, 7[$ (b) $] -7, \infty[$ (c) $] 7, \infty[$ (d) $] 7, \infty[$
- ⑥ What is the solution set of the inequality $3 \geq x > -2$ in R ?
 (a) $] -2, 3]$ (b) $] -2, 3[$ (c) $] -2, 3]$ (d) $] -2, 3[$
- ⑦ What is the solution set of the inequality $-x > 3$ in R ?
 (a) $] -\infty, -3[$ (b) $] -\infty, 3[$ (c) $] -3, \infty[$ (d) $] 3, \infty[$
- ⑧ What is the solution set of the inequality $5 > -x > -3$ in R ?
 (a) $] -5, 3]$ (b) $] -5, 3[$ (c) $] -3, 5]$ (d) $] -3, 5[$
- ⑨ The interval $[-1, 2[$ is the solution set in R for the inequality:
 (a) $-1 < x < 2$ (b) $-1 < x \leq 2$ (c) $-1 \leq x < 2$ (d) $-1 \leq x \leq 2$
- ⑩ If $-2 < 2x < 6$, then: $x \in \dots\dots\dots$
 (a) $] -1, 3[$ (b) $] -1, 3]$ (c) $] -1, 3[$ (d) $] -1, 3]$
- ⑪ Which of the following expresses the real values of x that satisfy: $-x < 7, x < 7$?
 (a) $7 > x > 0$ (b) $0 > x > -7$ (c) $7 > x > -7$ (d) $x < -7$

Second

Applying Concepts



► Find the solution set for each of the following equations in R :

- ⑫ $3x - \sqrt{3} = 5\sqrt{3}$
- ⑬ $3(x + 7) - 5x = x + 12$
- ⑭ $4 - \sqrt{6}x = |-10|$
- ⑮ $4(\sqrt{5}x + 8) + 20 = 72$

► Find the solution set for each of the following inequalities in R and represent it on the number line:

①⑥ $2X + 3 < 6$

①⑦ $5 - 3X \geq 2$

①⑧ $2(6 - 2X) \leq 8$

①⑨ $\frac{X}{2} - 3 > 7$

②⑩ $\frac{3X}{4} + \frac{X}{2} > 5$

②⑪ $25 \leq 5(3 - X)$

②② $2X + 9 \leq 6X - 1$

②③ $3X - 2 < 2(2 - X)$

► Find the real values of X that satisfy each of the following and represent it on the number line:

②④ $11 > 3X + 2 > -4$

②⑤ $-8 \leq 4 - 3(X + 1) < 10$

②⑥ $3X - 2 \leq 4(X - 1) \leq 3X + 6$

②⑦ $2X + 12 < 4(5 + X) < 3X + 23$

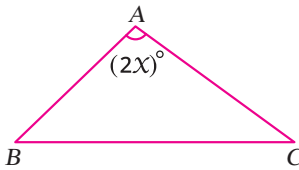
Third

Analysis and Subjects Integration:



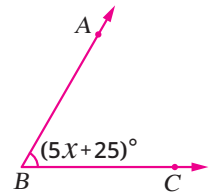
②⑧ **Geometry:**

In the given figure:
 ABC is an obtuse angled triangle at A , what are the possible values of X ?



②⑨ **Geometry:**

In the given figure:
If $\angle ABC$ is an acute angle, find the interval to which the value of X belongs.



③⑩ **Geometry:** A triangle with side lengths 9 cm , $(3X + 5)\text{ cm}$, 20 cm . What are the possible values of X ?

③⑪ **Car Rental:**

Two car rental companies have the following payment plans:
First company: 900 pounds per day, plus 1.5 pounds for each kilometer.
Second company: 1100 pounds per day, plus one pound for each kilometer.
What is the maximum number of kilometers that can be traveled in one day with the first company so that it costs less than the second company?



Honest competition between companies achieves better quality, and better price for the consumer.

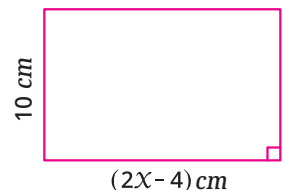
Creative Thinking



③⑫ **Geometry:**

In the given figure: A rectangle with dimensions 10 cm , $(2X - 4)\text{ cm}$.

- ① Find the value of X that makes the perimeter of the rectangle $= 52\text{ cm}$.
- ② Find the values of X that make the perimeter of the rectangle greater than 40 cm .
- ③ Find the values of X that make the area of the rectangle less than 80 cm^2 .



③⑬ If the solution set of the inequality $b \geq \frac{X-1}{2} \geq a$ is $[1, 7]$, find the value of $a \times b$.

Evaluate your understanding!

How well do you understand equations and inequalities of the first degree in one variable? Tick the right box



Unit Two Assessment

Evaluate your understanding!

How well do you understand Unit Two?
Tick the right box



► Choose the correct answer from the given ones:

- ① What is the domain of the function $\{(2, 5), (6, 7), (9, 7)\}$?
(a) $\{2, 6, 9\}$ (b) $\{5, 7\}$
(c) $\{2, 5, 6, 7, 9\}$ (d) $\{2, 5, 6\}$
- ② If: $n(X \times Y) = 12$, $n(X^2) = 16$ what is the value of $n(Y^2)$?
(a) 192 (b) 36
(c) 9 (d) 3
- ③ If the price of a car after t years of purchasing is given, in pounds by the function $f(t) = 1200000 - 50000t$ so after how many years does the price of the car become 800,000 pounds?
(a) 10 (b) 8 (c) 6 (d) 4
- ④ If $f(x) = 3$, what is the value of $\frac{2f(7)}{7f(2)}$?
(a) 1 (b) $\frac{6}{7}$
(c) $\frac{2}{7}$ (d) $\frac{7}{2}$

► Complete each of the following:

- ⑤ If $X = \{1, 2, 3\}$, $Y = \{12, 13\}$ and R is a function from X to Y where $R = \{(1, 12), (3, 13), (k, 12)\}$ then $k = \dots\dots\dots$
- ⑥ The solution set of the inequality $2X + 3 > 15$ in R is $\dots\dots\dots$
- ⑦ If $f(x) = 2x^2$, then $f(4) - f(2) = \dots\dots\dots$
- ⑧ The straight line representing the function f where $f(x) = 18 - 4x$ cuts the x -axis at the point $\dots\dots\dots$

► Answer the following questions:

- ⑨ Find the solution set of the inequality: $-3 < 2X + 5 < 13$ in R .
- ⑩ Represent graphically the function f where $f(x) = 3x - 1$
- ⑪ If $X = \{1, 3, 7\}$, $Y = \{5, 9\}$, find $X \times Y$, Y^2 .
- ⑫ If $X = \{0, 1, 2, 3\}$, $Y = \{6, 7, 8, 9, 10\}$ and R is a relation from X to Y where " XRY " means " $X + Y = 10$ " for every $X \in X$, $Y \in Y$ write the relation as a set of ordered pairs, then represent the relation with an arrow diagram stating the domain and the range of the relation. Does the relation represent a function or not? why?

Unit Two Activity Schedule and ordered pairs

► Activity Aim:

Employing the concept of ordered pairs in organizing elements and data related to life situations.

► Implementation Steps:

Using what was learned about the concept of ordered pairs, implement the following steps, then compare your answer with what one of your classmates reached:

- ① Using the weekly class schedule, express the following using ordered pairs.
 - Ordered pairs expressing (Day, Subject Name).
 - Ordered pairs expressing (Day, Math Class).
- ② Create a list of ordered pairs for any other subject you choose.
- ③ Compare your answer in step (1) with the answer of one of your classmates, and determine the extent of similarity and difference, with correction.
- ④ Choose a real life situations outside of school where the data can be expressed using ordered pairs such as: Family members (Name and Age), Market (Item and Price).



UNIT 3

Geometry

Unit Lessons

3-1 Axioms of Inequality

3-2 Inequality in Triangles

3-3 The Perpendicular Projections and Euclid's Theorem

3-4 The Circle

3-5 Right Circular Cylinder and Right Prism

3-6 The distance between Two Points in The Coordinate Plane

3-7 Slope of the Straight Line



Geometry theories are used in artificial intelligence applications, as artificial intelligence depends on Geometry theories to improve the accuracy of reaching a solution for some engineering problems.

► **Can Geometry theories be used to reach solutions for more complex problems in artificial intelligence applications?**

Issues and life skills

- Mathematical Communication
- Creative Thinking
- Critical Thinking
- Sustainable Development

Values

- Awareness of practicing sports
- Art Appreciation
- Work (Teamwork)

Lesson 3 - 1

Axioms of Inequality

■ Learning Outcomes

- Understand axioms of inequality.
- Use axioms of inequality in geometrical applications.

■ Vocabulary

- Axioms of inequality

■ Enriched Information

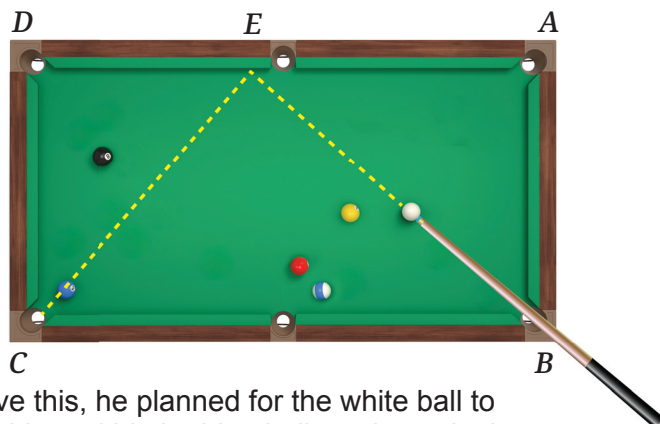
The axiom in mathematics is a statement assumed to be true based on fundamental principles. Such as: Euclid's axioms upon which Euclidean Geometry was built.

Get Ready!



Billiards is considered one of the games that combine entertainment, accuracy, precision, and concentration, it depends on mathematical concepts of angles and reflection.

A player wanted to hit the blue ball using the white ball, and to achieve this, he planned for the white ball to bounce off the side of the table and hit the blue ball as shown in the dashed path in the picture. If the area of $\triangle EDC <$ The area of $\triangle EAB$, is the area of $EDCB <$ The area of $EABC$?



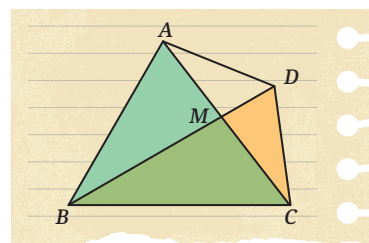
Think & Discuss!



In the opposite figure:

If $m(\angle BAC) = m(\angle BDC)$,
 $m(\angle ADC) > m(\angle BAD)$

Compare between $m(\angle CAD)$ and $m(\angle BDA)$, Discuss.



Learn!



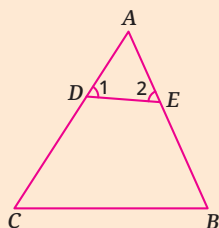
Axioms of inequality

If a, b, c are three quantities, then:

Addition	If: $a < b$ then: $a + c < b + c$
Subtraction	If: $a < b$ then: $a - c < b - c$
Multiplication by a positive number	If: $a < b$, c is a positive number then: $a \times c < b \times c$
Transitive relation	If: $a < b$, $b < c$ then: $a < c$
Summation	If: $a < b$, $c < d$ then: $a + c < b + d$
Comparison between the whole and the part:	If: $a = b + c$ where c is a positive number then: $a > b$

■ Self-Evaluation ①

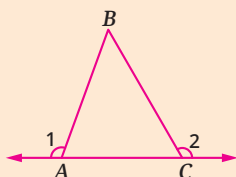
In the following figure:



$AC > AB$,
 $m(\angle 1) = m(\angle 2)$
 Prove that: $DC > EB$

■ Self-Evaluation ②

In the following figure:



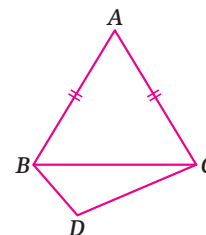
Prove that:
 $m(\angle 1) + m(\angle 2) > 180^\circ$

Example 1

In the given figure:

$AB = AC$, $m(\angle DBC) > m(\angle DCB)$

Prove that: $m(\angle ABD) > m(\angle ACD)$



$\therefore AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB)$ (1)

$\therefore m(\angle DBC) > m(\angle DCB)$ (2)

From (1), (2) we conclude that:

$m(\angle DBC) + m(\angle ABC) > m(\angle DCB) + m(\angle ACB)$

$\therefore m(\angle ABD) > m(\angle ACD)$

Note

The measure of the exterior angle of a triangle is greater than the measure of any interior angle except the adjacent one.

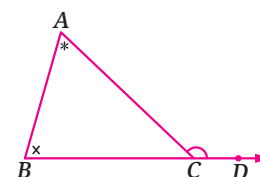
In the given figure:

ABC is a triangle, $D \in \overrightarrow{BC}$, then $\angle ACD$ is exterior to $\triangle ABC$

$\therefore m(\angle ACD) = m(\angle A) + m(\angle B)$

And from the axiom of comparison between the whole and the part we conclude that:

$m(\angle ACD) > m(\angle A)$, $m(\angle ACD) > m(\angle B)$

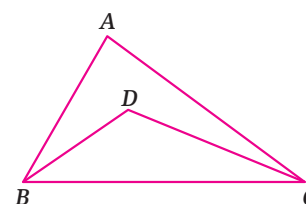


Example 2

In the given figure:

The point D lies inside the triangle ABC

Prove that: $m(\angle BDC) > m(\angle A)$



Construction: Draw $\overrightarrow{CD}$ to intersect $\overline{AB}$ at E

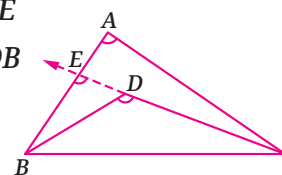
Proof: $\therefore \angle BDC$ is an exterior angle of $\triangle EDB$

$\therefore m(\angle BDC) > m(\angle BED)$ (1)

$\therefore \angle BED$ is an exterior angle of $\triangle ACE$

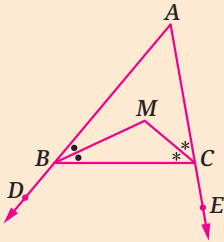
$\therefore m(\angle BED) > m(\angle A)$ (2)

From (1), (2) we conclude that: $m(\angle BDC) > m(\angle A)$



■ Self-Evaluation ③

In the following figure:



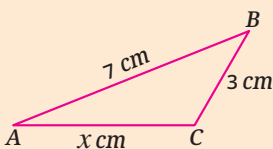
$$m(\angle ABM) < m(\angle ACM)$$

Prove that:

$$m(\angle DBC) > m(\angle ECB)$$

■ Self-Evaluation ④

In the following figure:



Find the set of possible values of x

Note :

- The complement of the smaller angle in measure is greater than the complement of the larger angle in measure.
- The supplement of the smaller angle in measure is greater than the supplement of the larger angle in measure.

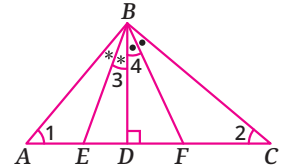
Example 3

In the given figure:

ABC is a triangle, $D \in \overline{AC}$ such that $\overline{BD} \perp \overline{AC}$,
 $\overrightarrow{BE}$ bisects $\angle ABD$, $\overrightarrow{BF}$ bisects $\angle CBD$

If $m(\angle 1) > m(\angle 2)$

Prove that: $m(\angle 3) < m(\angle 4)$



$$\therefore \overline{BD} \perp \overline{AC}$$

$$\therefore m(\angle ADB) = m(\angle CDB) = 90^\circ$$

$$\therefore \angle 1 \text{ complements } \angle ABD, \angle 2 \text{ complements } \angle CBD$$

$$\therefore m(\angle 1) > m(\angle 2)$$

$$\therefore m(\angle ABD) < m(\angle CBD)$$

$$\therefore \frac{1}{2} m(\angle ABD) < \frac{1}{2} m(\angle CBD) \quad \therefore m(\angle 3) < m(\angle 4)$$

Previous Knowledge: Triangle Inequality

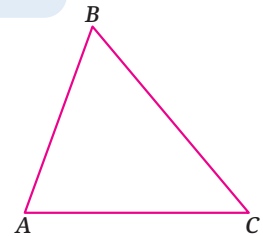
The sum of the lengths of any two sides in a triangle is greater than the length of the third side.

In $\triangle ABC$:

$$AB + BC > AC$$

$$AC + CB > AB$$

$$AC + AB > BC$$



Note :

The length of any side in a triangle is greater than the difference between the lengths of the other two sides and less than their sum.

$$|AC - AB| < BC < AC + AB$$

Example 4

In the given figure:

If $D \in \overline{BC}$, $AD = DC$

Prove that: $BC > AB$



In $\triangle ADB$:

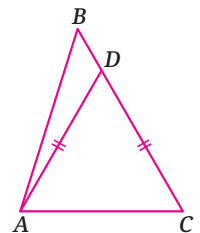
$$AD + DB > AB$$

(Triangle inequality)

$$\therefore AD = DC$$

$$\therefore CD + DB > AB$$

$$\therefore BC > AB$$



Lesson Assessment



First

Measuring Conceptual Understanding



► Choose the correct answer from the given ones:

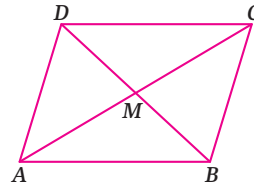
① In the given figure:



Which of the following statements is true ?

- (a) $AB < CD$
- (b) $AC < BD$
- (c) $AC > BD$
- (d) $AC = BD$

② In the given figure:

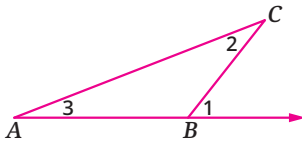


$ABCD$ is a parallelogram, $AC > BD$

Which of the following is correct?

- (a) $MC < MD$
- (b) $MC = MD$
- (c) $MC < MB$
- (d) $MC > MB$

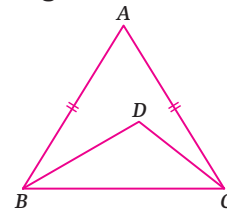
③ In the given figure:



If $m(\angle 2) > m(\angle 3)$,
which of the following is true ?

- (a) $m(\angle 1) > m(\angle 2) > m(\angle 3)$
- (b) $m(\angle 2) > m(\angle 3) > m(\angle 1)$
- (c) $m(\angle 2) > m(\angle 1) > m(\angle 3)$
- (d) $m(\angle 1) > m(\angle 3) > m(\angle 2)$

④ In the given figure:



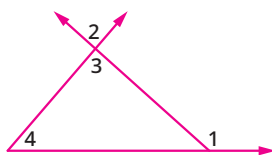
$m(\angle DBC) < m(\angle DCB)$, $AB = AC$

Which of the following is true ?

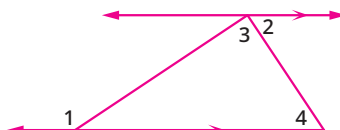
- (a) $m(\angle ABC) < m(\angle ACB)$
- (b) $m(\angle ACD) < m(\angle ABD)$
- (c) $m(\angle ACD) > m(\angle ABD)$
- (d) $m(\angle ACD) = m(\angle ABD)$

► In each of the following figures, explain why $m(\angle 1) > m(\angle 2)$:

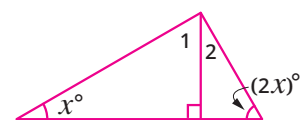
⑤



⑥



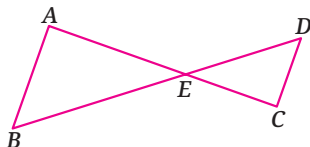
⑦





► Choose the correct answer from the given ones:

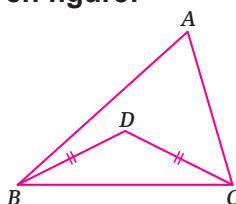
8 In the given figure:



$$BE > AE, ED > EC$$

Prove that: $BD > AC$

10 In the given figure:



$$DB = DC, m(\angle ABC) < m(\angle ACB)$$

Prove that: $m(\angle ACD) > m(\angle ABD)$

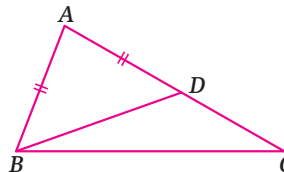
12 In the given figure:

$ABCD$ is a parallelogram,

$$E \in \overrightarrow{BC}$$

Prove that: $m(\angle DCE) > m(\angle ADB)$

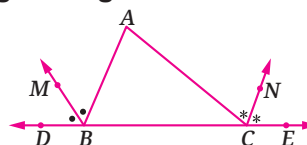
9 In the given figure:



$$C \in \overrightarrow{AD}, AB = AD$$

Prove that: $m(\angle ABD) > m(\angle C)$

11 In the given figure:

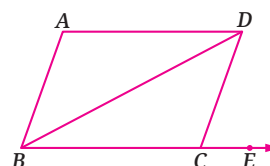


$$m(\angle ABC) > m(\angle ACB),$$

$\overrightarrow{BM}$ bisects $\angle ABD$,

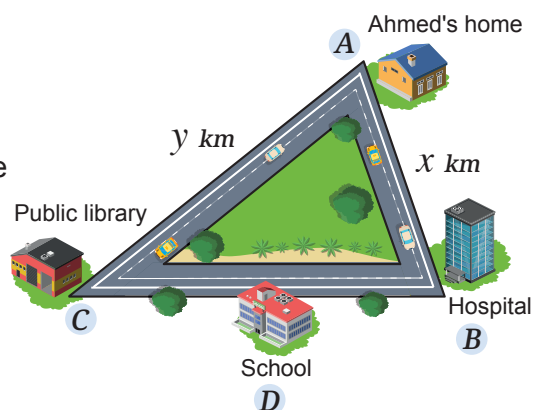
$\overrightarrow{CN}$ bisects $\angle ACE$,

Prove that: $m(\angle ABM) < m(\angle ACN)$



13 In the given figure:

If the school is located at the midpoint of the distance between the public library and the hospital, and $3X = 2Y$ Ahmed wanted to go from his home to the school such that he takes the shortest route. Determine the path Ahmed can take.

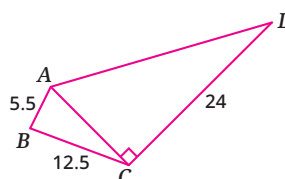


Creative Thinking



14 In the given figure:

Find the set of possible values for the length of each of $\overline{AD}$, $\overline{AC}$



Evaluate your understanding!

How well do you understand Axioms of inequality? Tick the right box



Lesson 3 - 2

Inequality in Triangles

■ Learning Outcomes

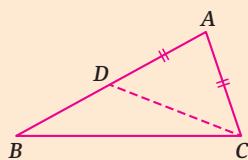
- Understand inequality theorems in a triangle.
- Understand inequality theorems in two triangles.
- Use inequality theorems in triangles to solve some real life problems.

■ Vocabulary

- Inequality

■ Discuss (Theorem proof)

Take a point D on $\overline{AB}$ such that $AD = AC$, then draw $\overline{DC}$

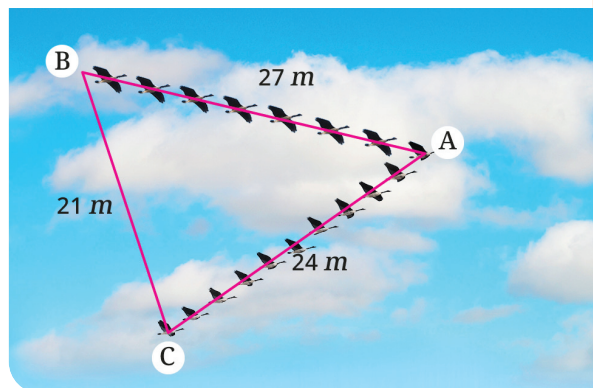


Then use the properties of the isosceles triangle ADC and the exterior angle of $\triangle BDC$ to prove that:
 $m(\angle ACB) > m(\angle B)$

Get Ready!



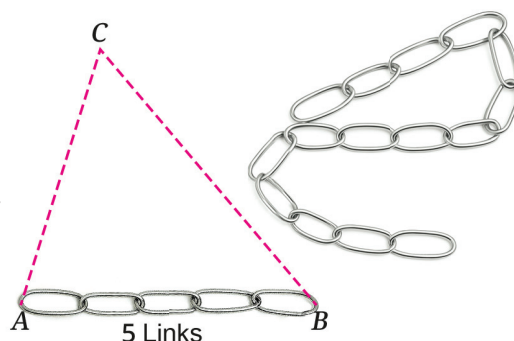
Bird migration is considered one of the most amazing natural phenomena in the animal world, as they move every year from certain places to others in search of food and warmth. Some birds fly thousands of kilometers in long journeys across seas and mountains, and these journeys show amazing ability of endurance and accuracy of organization. The picture in front of you represents a flock of migrating birds. Does the angle at the front of the flock have the largest measure? Explain what you say.



Think & Discuss!



Sarah wanted to form triangle ABC using a chain consisting of 15 links of equal length such that the triangle is scalene, and $m(\angle A) > m(\angle B)$. What is the appropriate number of links for each of the two sides $\overline{AC}$, $\overline{CB}$? Discuss the different possibilities.



Learn!

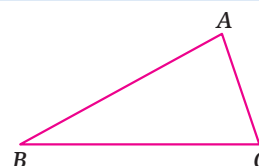


Inequality in the triangle

Theorem

If two sides of a triangle are unequal in length, then the longer side is opposite an angle that is greater in measure than the angle opposite the other side.

In $\triangle ABC$:
If $AB > AC$
Then $m(\angle C) > m(\angle B)$



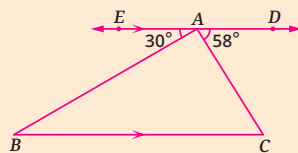
■ Discuss with your classmates

Which of the triangle sides is longer in length if:

- ① The triangle is right-angled?
- ② The triangle is obtuse-angled?

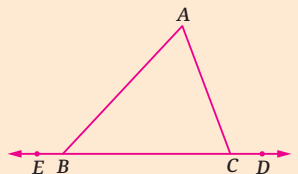
■ Self-Evaluation ①

- ① In the following figure:



Prove that: $AB > AC$

- ② In the following figure:



If $AB > AC$ prove that:
 $m(\angle ABE) > m(\angle ACD)$

For Example:

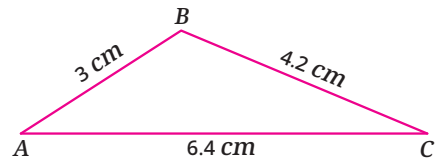
In the given figure:

$m(\angle B) > m(\angle A)$ Because: $AC > BC$

$m(\angle A) > m(\angle C)$ Because: $BC > BA$

Meaning that:

$m(\angle B) > m(\angle A) > m(\angle C)$



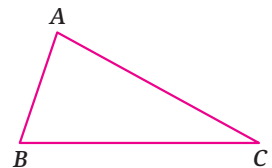
Converse of the Theorem

If two angles of a triangle are unequal in measure, then the greater angle in measure is opposite the longer side than the side opposite the other angle.

In $\triangle ABC$:

If $m(\angle B) > m(\angle C)$

Then $AC > AB$



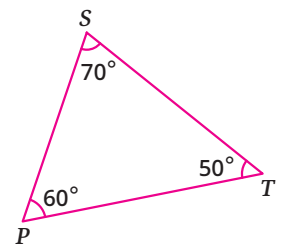
For Example:

In the given figure:

$PT > ST$ because: $m(\angle S) > m(\angle P)$,

$ST > SP$ because: $m(\angle P) > m(\angle T)$

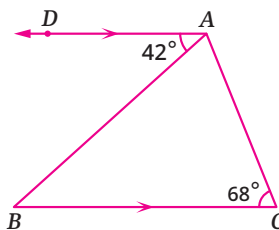
Meaning that: $PT > ST > SP$



Example 1

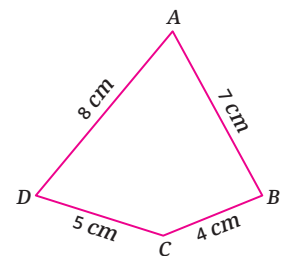
Prove what is required below each figure:

①



Prove that: $BC > AC$

②



Prove that: $m(\angle ABC) > m(\angle ADC)$



①

$\therefore \overline{CB} \parallel \overline{AD}$

$\therefore m(\angle B) = m(\angle DAB) = 42^\circ$ (Two alternating interior angles)

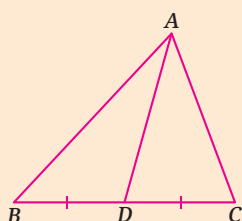
$\therefore m(\angle BAC) = 180^\circ - (42^\circ + 68^\circ) = 70^\circ$

$\therefore m(\angle BAC) > m(\angle B)$

$\therefore BC > AC$

■ Self-Evaluation ②

In the following figure:



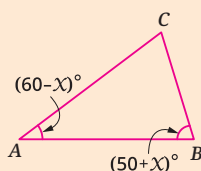
$\overline{AD}$ is a median in $\triangle ABC$,
 $BC < 2AD$

Prove that:

$$m(\angle BAC) < 90^\circ$$

■ Self-Evaluation ③

In the following figure:



Find the set of values of x
 satisfy that $AC > BC$

② **Construction:** Draw $\overline{BD}$

Proof: In $\triangle ABD$

$$\because AD > AB \quad \therefore m(\angle 1) > m(\angle 2) \dots (1)$$

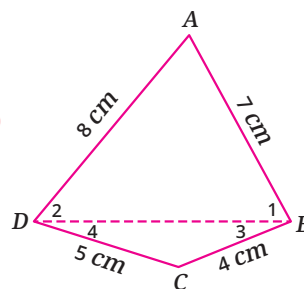
In $\triangle CBD$:

$$\because DC > CB \quad \therefore m(\angle 3) > m(\angle 4) \dots (2)$$

By adding (1), (2)

$$\therefore m(\angle 1) + m(\angle 3) > m(\angle 2) + m(\angle 4)$$

$$\therefore m(\angle ABC) > m(\angle ADC)$$

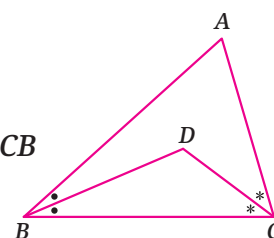


Example 2

In the given figure:

$AB > AC$, $\overrightarrow{BD}$ bisects $\angle ABC$ and $\overrightarrow{CD}$ bisects $\angle ACB$

Prove that: $BD > CD$



$\because \overrightarrow{BD}$ bisects $\angle ABC$

$$\therefore m(\angle DBC) = \frac{1}{2} m(\angle ABC)$$

$\because \overrightarrow{CD}$ bisects $\angle ACB$

$$\therefore m(\angle DCB) = \frac{1}{2} m(\angle ACB)$$

In $\triangle ABC$:

$\because AB > AC$

$$\therefore m(\angle ACB) > m(\angle ABC) \quad \therefore \frac{1}{2} m(\angle ACB) > \frac{1}{2} m(\angle ABC)$$

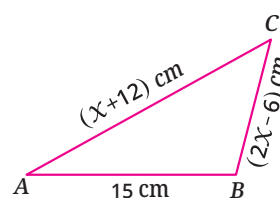
$$\therefore m(\angle DCB) > m(\angle DBC) \quad \therefore BD > CD$$

Example 3

In the given figure:

Find the set of values of x

that satisfy that $m(\angle A) < m(\angle B)$



In $\triangle ABC$:

$\because AC + BC > 15$ (Triangle Inequality)

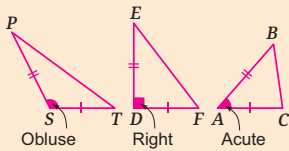
$$\therefore x + 12 + 2x - 6 > 15 \quad \therefore 3x + 6 > 15 \quad \therefore 3x > 15 - 6$$

$$\therefore 3x > 9 \quad \therefore x > \frac{9}{3} \quad \therefore x > 3$$

$$\because m(\angle A) < m(\angle B) \quad \therefore BC < AC \quad \therefore 2x - 6 < x + 12$$

$$\therefore 2x - x < 12 + 6 \quad \therefore x < 18 \quad \therefore 3 < x < 18$$

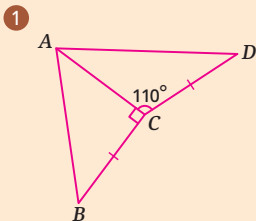
■ Note that



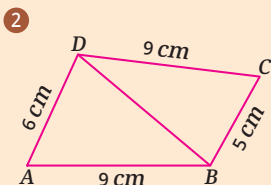
$$BC < EF < PT$$

The length of the side opposite the angle included between two sides in a triangle increases as the measure of this angle increases.

■ Self-Evaluation ④



Compare between the length of $\overline{AD}$ and the length of $\overline{AB}$



Compare between $m(\angle CDB)$, $m(\angle ABD)$

Inequality in two triangles

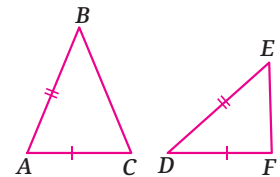
Theorem

If two sides in a triangle are equal in length to the corresponding two sides in another triangle, and the measures of the included angles between the two sides differ, then the greater angle in measure is opposite to the longer side in length.

$$\text{If } AC = DF, AB = DE,$$

$$m(\angle A) > m(\angle D)$$

$$\text{Then } BC > EF$$

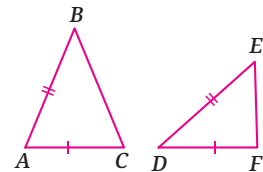


Converse of the theorem

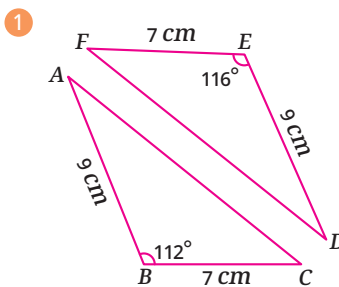
If two sides in a triangle are equal in length to the corresponding two sides in another triangle, and the lengths of the third sides differ, then the longer side in length is opposite to the angle with the greater measure.

$$\text{If } AC = DF, AB = DE, BC > EF$$

$$\text{Then } m(\angle A) > m(\angle D)$$



Example 4



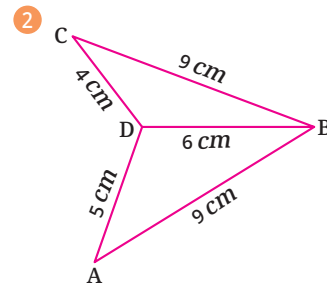
Compare between the length of $\overline{AC}$ and the length of $\overline{FD}$



$$\textcircled{1} \because BC = EF, AB = ED$$

$$\therefore m(\angle B) < m(\angle E)$$

$$\therefore AC < FD$$



Compare between $m(\angle ABD)$, $m(\angle CBD)$

$$\textcircled{2} \because AB = CB, DB = DB$$

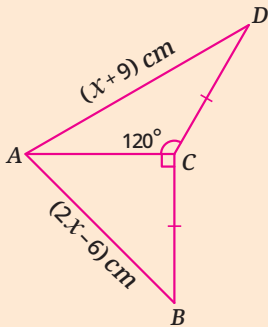
(Common side)

$$\therefore AD > CD$$

$$\therefore m(\angle ABD) > m(\angle CBD)$$

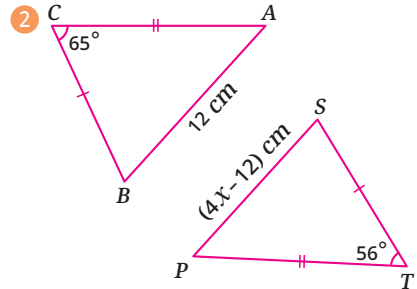
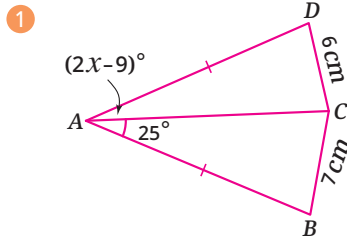
■ Self-Evaluation ⑤

Find the set of possible values for x in the following figure:



Example 5

Find the set of possible values for x in each of the following:



- ① $\because AB = AD$
 $AC = AC$ (Common side)
 $DC < BC$
 $\therefore m(\angle DAC) < m(\angle BAC)$
 $\therefore 2x - 9 < 25$
 $\therefore 2x < 25 + 9$
 $\therefore 2x < 34 \quad \therefore x < 17$
 $\because m(\angle DAC) > 0$
 $\therefore 2x - 9 > 0 \quad \therefore 2x > 9$
 $\therefore x > 4.5$
 $\therefore 4.5 < x < 17$

- ② $\because AC = PT, BC = ST$

$$m(\angle T) < m(\angle C)$$

$$\therefore PS < AB$$

$$\therefore 4x - 12 < 12$$

$$\therefore 4x < 12 + 12$$

$$\therefore 4x < 24 \quad \therefore x < 6$$

$$\because PS > 0$$

$$\therefore 4x - 12 > 0$$

$$\therefore 4x > 12 \quad \therefore x > 3$$

$$\therefore 3 < x < 6$$

Lesson Assessment



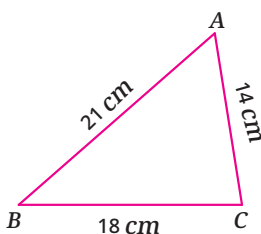
First

Measuring Conceptual Understanding

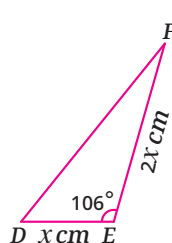


- ① Arrange in ascending order the measures of angles in each of the following triangles:

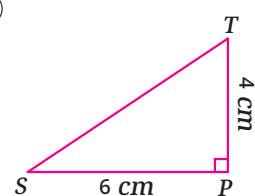
①



②

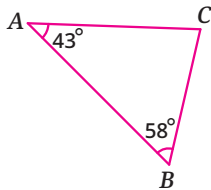


③

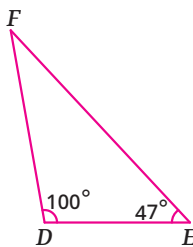


② Arrange in ascending order the side lengths in each of the following triangles:

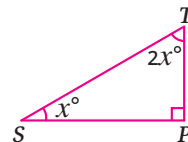
①



②

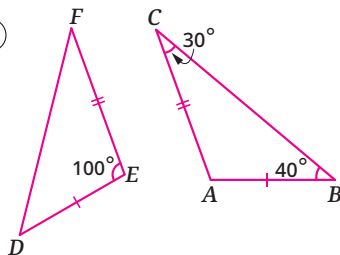


③



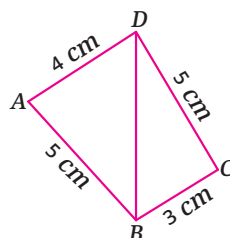
③ Compare between the required measures below each figure:

①



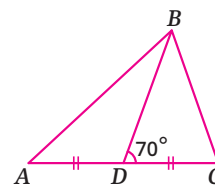
The length of $\overline{FD}$ and the length of $\overline{CB}$

②



$m(\angle CDB)$, $m(\angle ABD)$

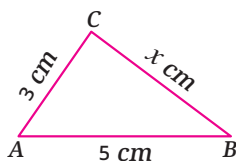
③



The length of $\overline{AB}$ and the length of $\overline{BC}$

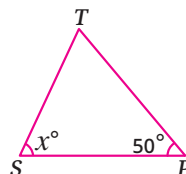
④ Find the possible values of x that satisfy the condition below each figure:

①



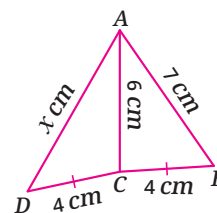
$m(\angle A) > m(\angle B)$

②



$TP > ST$

③



$AD > AB$

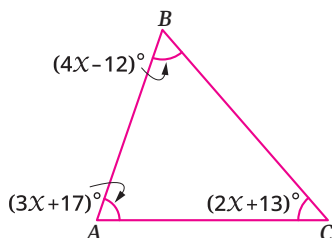
Second

Applying Concepts

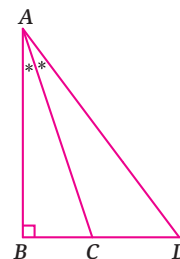


⑤ In each of the two following figures, arrange the side lengths of triangle ABC from smallest to largest:

①



②

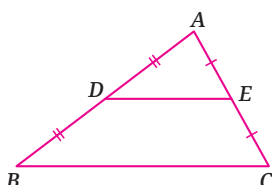


⑥ In the given figure:

If $AD > AE$

Prove that:

$m(\angle C) > m(\angle B)$

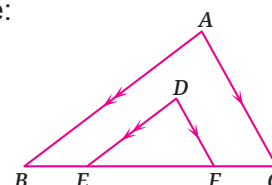


⑦ In the given figure:

If $AC < AB$

Prove that:

$DE > DF$

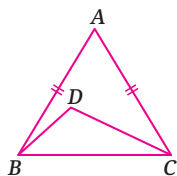


8 In the given figure:

$$AB = AC$$

$$m(\angle ACD) > m(\angle ABD)$$

Prove that: $DC > DB$



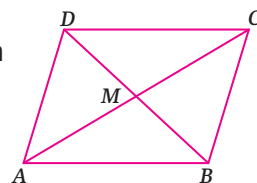
9 In the given figure:

$ABCD$ is a parallelogram
Its diagonals intersect

at M , $AB > BC$

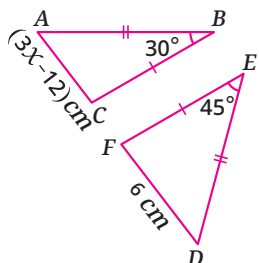
Prove that:

$$m(\angle AMB) > m(\angle CMB)$$

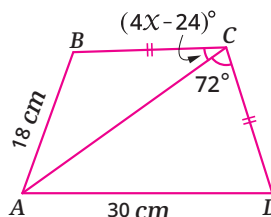


10 Find the possible values of X in each of the following:

①



②



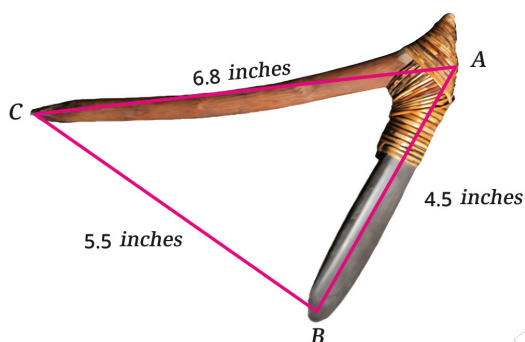
Third

Analysis and Subjects Integration

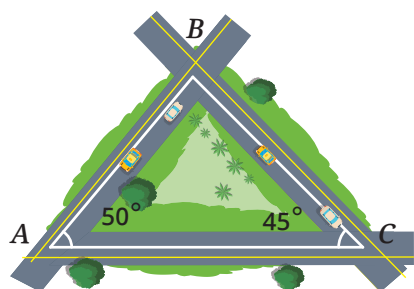


11 Archaeology :

The Ancient Egyptians used the opposite tool to smooth and shape wooden pieces. Which of the two angles is larger? Is it ($\angle A$), which is the angle formed by the blade of the tool with the handle, or ($\angle B$), which is the angle formed by the blade of the tool with the work surface? Explain that.



The Ancient Egyptians excelled in handicrafts, so they mastered making jewelry, woodworking, metalwork, engraving, and building with amazing accuracy and creativity



12 Intersection of roads :

Two roads meet at an angle of measure 50° at point A . A third road from B to C makes an angle of measure 45° with the road from A to C .

Which of the two intersections A or B is closer to C ? Explain.

Creative Thinking

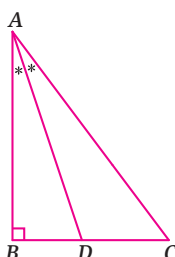


13 In the opposite figure:

$\triangle ABC$ is a right-angled triangle at B ,

$\overrightarrow{AD}$ bisects $\angle BAC$,

Prove that: $DC > DB$



Evaluate your understanding!

How well do you understand Inequality in Triangles? Tick the right box



Lesson 3 - 3

The Perpendicular Projections and Euclid's Theorem

■ Learning Outcomes

- Understand the perpendicular projection of a point on a straight line.
- Understand the perpendicular projection of a line segment on a straight line.
- Understand the Euclid's theorem and its results.
- Use Euclid's theorem in solving some real life problems.

■ Vocabulary

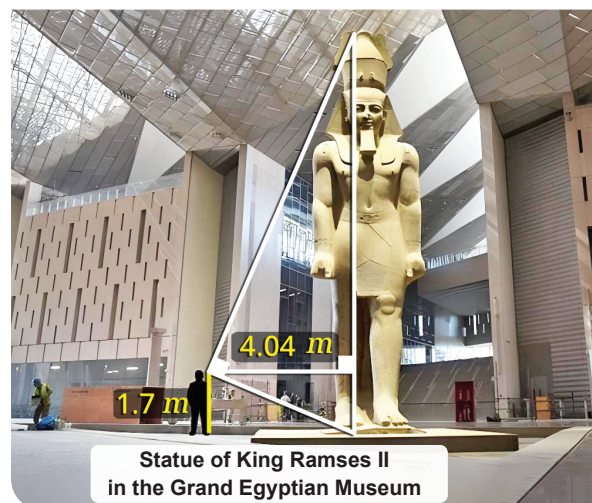
- The perpendicular projection
- Euclid's theorem
- Geometric mean

Get Ready!



The statue of King Ramses II is a unique masterpiece that embodies the greatness of the Pharaohs, and the creativity of ancient Egyptians in the art of sculpture.

It was discovered in 1820 AD near Mit Rahina south of Giza, then it was moved to Bab Al-Hadid Square in 1955 AD to change the square's name to Ramses Square, and remained there until it was moved to its current location in the grand Egyptian museum. Can you determine the height of the statue from the data shown in the picture?



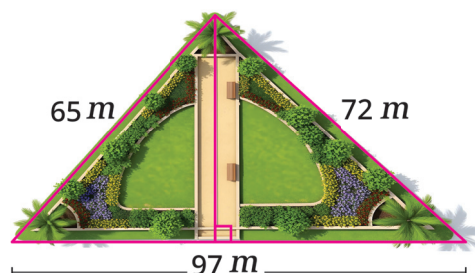
Statue of King Ramses II
in the Grand Egyptian Museum

In this lesson, you will learn about the perpendicular projection and Euclid's theorem, which will enable you to solve such real life problems.

Think & Discuss!



Hamed has a triangular piece of land with dimensions 65 m, 72 m, 97 m. He wants to divide it with a path perpendicular to its longest side. Can you know the length of this path?



Learn!



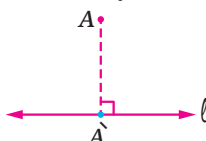
The perpendicular projections

Projection of a point on a straight line

The projection of a point on a straight line is the position of the perpendicular dropped from the point onto this straight line

If $A \notin \ell$ then:

The projection of point A on the straight line ℓ is the point A' where $\overline{AA'} \perp \ell$



If $A \in \ell$ then:

The projection of point A on the straight line ℓ is the point A itself.



■ Discuss with your classmates

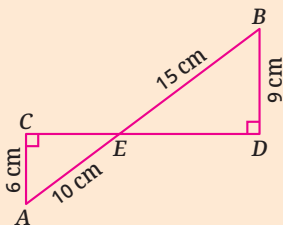
What is the relation between the length of the line segment and the length of its projection on a given straight line in each of the following cases:

- 1 The line segment is parallel to the straight line.
- 2 The line segment is not parallel to the straight line.
- 3 The line segment is perpendicular to the straight line.

Can the length of the projection of the line segment on a straight line be greater than the length of the line segment itself?

■ Self-Evaluation ①

In the following figure:

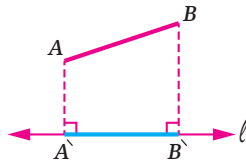


Find the length of each of the following:

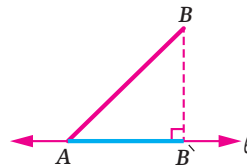
- 1 The projection of $\overline{AE}$ on $\overleftrightarrow{CD}$
- 2 The projection of $\overline{BE}$ on $\overleftrightarrow{CD}$
- 3 The projection of $\overline{AB}$ on $\overleftrightarrow{CD}$

Projection of a line segment on a straight line

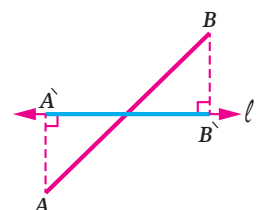
The projection of a line segment on a straight line is the line segment whose endpoints are the projections of the endpoints of the original line segment on this straight line.



The projection of $\overline{AB}$ on the straight line ℓ is $\overline{A'B'}$



The projection of $\overline{AB}$ on the straight line ℓ is $\overline{A'B'}$



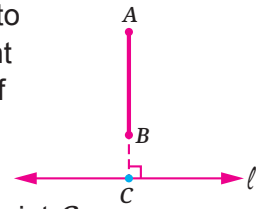
The projection of $\overline{AB}$ on the straight line ℓ is $\overline{A'B'}$

Note

If the line segment is in a perpendicular position to the straight line, then its projection on this straight line is one point which is the projection of each of its endpoints.

In the given figure:

The projection of $\overline{AB}$ on the straight line ℓ is the point C

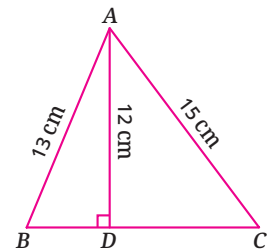


Example ①

In the given figure:

Find the length of each of the following:

- 1) The projection of $\overline{AC}$ on $\overleftrightarrow{BC}$
- 2) The projection of $\overline{AB}$ on $\overleftrightarrow{BC}$



$$\therefore \overline{AD} \perp \overleftrightarrow{BC}, D \in \overleftrightarrow{BC}$$

$\therefore \overline{DC}$ is the projection of $\overline{AC}$ on $\overleftrightarrow{BC}$,
 $\overline{DB}$ is the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$

$\therefore \triangle ADC$ is right-angled at D:

$$\therefore DC = \sqrt{(15)^2 - (12)^2} = \sqrt{81} = 9$$

$\therefore$ The length of the projection of $\overline{AC}$ on $\overleftrightarrow{BC}$ equals 9 cm

$\therefore \triangle ADB$ is right-angled at D

$$\therefore DB = \sqrt{(13)^2 - (12)^2} = \sqrt{25} = 5$$

$\therefore$ The length of the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$ equals 5 cm

■ Previous Knowledge

Pythagoras theorem:

If $\triangle ABC$ is right-angled at B

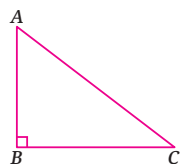
Then:

$$(\overline{AC})^2 = (\overline{AB})^2 + (\overline{BC})^2$$

and:

$$(\overline{AB})^2 = (\overline{AC})^2 - (\overline{BC})^2$$

$$(\overline{BC})^2 = (\overline{AC})^2 - (\overline{AB})^2$$



History of Mathematics

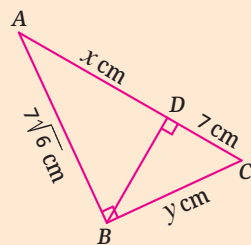
Euclid is a Greek mathematician who lived in the city of Alexandria around 300 BC, nicknamed the "father of geometry".

He is famous for his famous book "Elements" which is considered one of the most important books in the history of mathematics



Self-Evaluation 2

In the following figure, find the value of each of x , y



Note that

In the given figure:

$$\therefore (AD)^2 = DB \times DC$$

then AD is the positive geometric mean between DB , DC

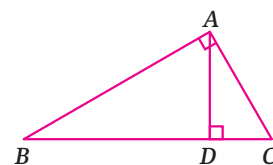
Euclid's theorem

The area of the square drawn on one of the two sides of the right angle in the right-angled triangle equals the area of the rectangle whose dimensions are the length of the projection of this side on the hypotenuse, and the length of the hypotenuse.

If $\triangle ABC$ is right-angled at A ,

$D \in \overline{BC}$ such that $\overline{AD} \perp \overline{BC}$ then:

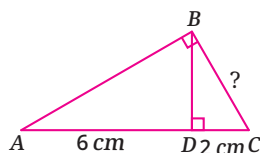
$$(AC)^2 = CD \times CB, \quad (AB)^2 = BD \times BC$$



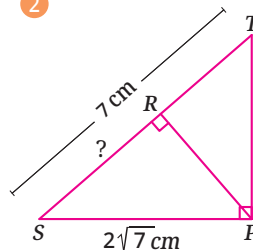
Example 2

In each of the following figures, find the side length indicated by the mark (?).

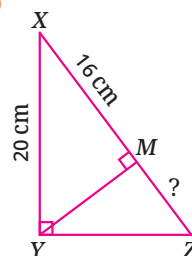
1



2



3



$$\textcircled{1} \therefore (BC)^2 = CD \times CA \quad (\text{Euclid's theorem})$$

$$\therefore (BC)^2 = 2 \times (2 + 6) = 16$$

$$\therefore BC = \sqrt{16} \quad \therefore BC = 4$$

Meaning that: The length of $\overline{BC}$ equals 4 cm.

$$\textcircled{2} \therefore (SP)^2 = SR \times ST \quad (\text{Euclid's theorem})$$

$$\therefore (2\sqrt{7})^2 = SR \times 7$$

$$\therefore 28 = SR \times 7 \quad \therefore SR = \frac{28}{7} = 4$$

Meaning that: The length of $\overline{SR}$ equals 4 cm.

$$\textcircled{3} \therefore (XY)^2 = XM \times XZ \quad (\text{Euclid's theorem})$$

$$\therefore (20)^2 = 16(16 + ZM) \quad \therefore 400 = 16(16 + ZM)$$

$$\therefore 16 + ZM = \frac{400}{16} \quad \therefore 16 + ZM = 25 \quad \therefore ZM = 25 - 16 = 9$$

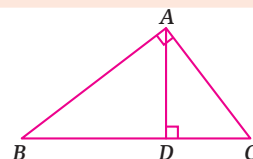
Meaning that: The length of $\overline{ZM}$ equals 9 cm.

Result:

If $\triangle ABC$ is a right-angled triangle at A ,

$D \in \overline{BC}$ such that $\overline{AD} \perp \overline{BC}$

Then: $(AD)^2 = DB \times DC$



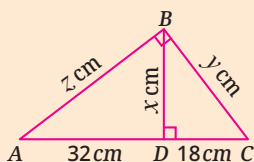
Note : In the previous figure: $AD = \frac{AB \times AC}{BC}$

"You can prove this note using the triangle area law"

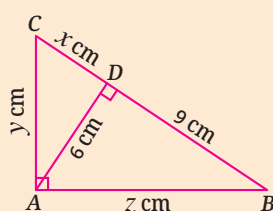
■ Self-Evaluation ③

In each of the two following figures find values of x , y , z :

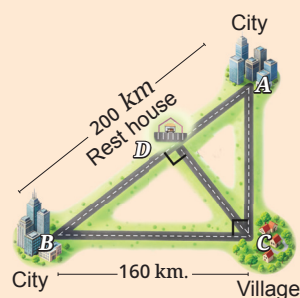
1



2



■ Self-Evaluation ④



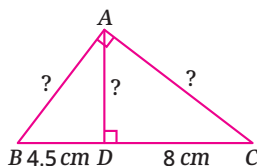
The drawn figure is a proposed diagram for building a rest house D on the highway between two cities A , B .

What is the length of the road connecting this rest house to the village C ?

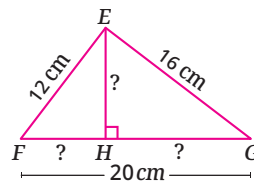
Example 3

In each of the two following figures, find the side lengths indicated by the mark (?).

1



2



$$\begin{aligned} ① \because (AD)^2 &= DB \times DC \\ &= 4.5 \times 8 = 36 \\ \therefore AD &= \sqrt{36} = 6 \\ \therefore (AC)^2 &= CD \times CB \\ &= 8 \times 12.5 = 100 \\ \therefore AC &= \sqrt{100} = 10 \\ \therefore (AB)^2 &= BD \times BC \\ &= 4.5 \times 12.5 = 56.25 \\ \therefore AB &= \sqrt{56.25} = 7.5 \text{ cm.} \end{aligned}$$

$$\begin{aligned} ② \text{ In } \triangle EFG : \\ \because (20)^2 &= (12)^2 + (16)^2 \\ \therefore m(\angle FEG) &= 90^\circ \\ \therefore EH &= \frac{EF \times EG}{FG} \\ &= \frac{12 \times 16}{20} = 9.6 \\ \therefore (EG)^2 &= GH \times GF \\ \therefore (16)^2 &= GH \times 20 \\ \therefore GH &= \frac{(16)^2}{20} = 12.8 \text{ cm.} \\ \therefore FH &= 20 - 12.8 = 7.2 \text{ cm.} \end{aligned}$$

Example 4

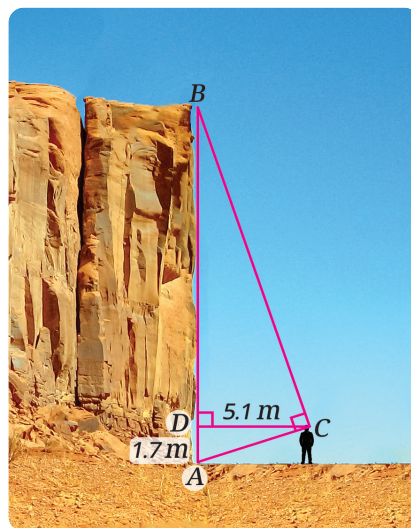
Link to life:

If you know that the height of the man is 1.7 meters, and he is at a distance of 5.1 meters from the rock. Find the height of the rock (Length of $\overline{AB}$)



$$\begin{aligned} \therefore m(\angle ACB) &= 90^\circ, \overline{CD} \perp \overline{AB} \\ \therefore (CD)^2 &= AD \times BD \\ \therefore (5.1)^2 &= 1.7 \times BD \\ \therefore BD &= \frac{(5.1)^2}{1.7} = 15.3 \\ \therefore AB &= 1.7 + 15.3 = 17 \end{aligned}$$

$\therefore$ The height of the rock is 17 meters.



■ Self-Evaluation ⑤

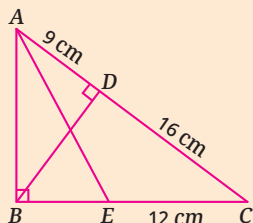
In the following figure:

$\triangle ABC$ is right-angled at B ,

$\overline{BD} \perp \overline{AC}$,

$AD = 9 \text{ cm}$, $CD = 16 \text{ cm}$,
 $EC = 12 \text{ cm}$

Find the length of $\overline{AE}$



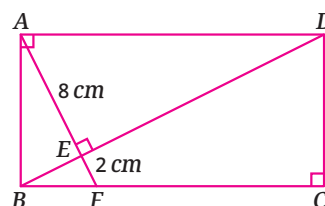
Example 5

In the given figure:

$ABCD$ is a rectangle, $F \in \overline{BC}$, $\overline{AF} \perp \overline{BD}$

$AE = 8 \text{ cm}$, $EF = 2 \text{ cm}$

Find the length of $\overline{BD}$



$\therefore \triangle ABF$ is right-angled at B , $\overline{BE} \perp \overline{AF}$

$$\therefore (BE)^2 = EA \times EF = 8 \times 2 = 16 \quad \therefore BE = 4$$

$\therefore \triangle BAD$ is right-angled at A , $\overline{AE} \perp \overline{BD}$

$$\therefore (AE)^2 = EB \times ED \quad \therefore 64 = 4 \times ED \quad \therefore ED = 16$$

$$\therefore BD = BE + ED = 4 + 16 = 20$$

Meaning that: The length of $\overline{BD}$ equals 20 cm

Collaborative Activity



Use different sources (such as: the Internet, your school library or ...) and cooperate with your classmates under supervision of your teacher to learn more about the Mathematician Euclid and his axioms upon which he built his geometric system.

Lesson Assessment



First

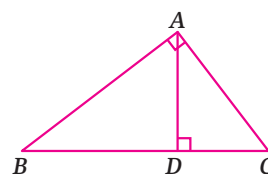
Measuring Conceptual Understanding



► Choose the correct answer from the given ones:

① In the given figure:

$\triangle ABC$ is right-angled at A , $D \in \overline{BC}$ such that $\overline{AD} \perp \overline{BC}$



① What is the projection of $\overline{AB}$ on $\overleftarrow{BC}$?

(a) $\overline{AD}$

(b) $\overline{DB}$

(c) $\overline{DC}$

(d) $\{D\}$

② What is the projection of $\overline{AC}$ on $\overleftarrow{BC}$?

(a) $\overline{AD}$

(b) $\overline{DB}$

③ What is the projection of $\overline{AD}$ on $\overleftarrow{BC}$?

(a) $\overline{AD}$

(b) $\overline{DB}$

(c) $\overline{DC}$

(d) $\{D\}$

(c) $\overline{DC}$

(d) $\{D\}$

④ What is the projection of $\overline{BC}$ on $\overleftarrow{AB}$?

(a) $\overline{BC}$

(b) $\overline{AC}$

⑤ What is the projection of $\overline{CB}$ on $\overleftarrow{AC}$?

(a) $\overline{BC}$

(b) $\overline{CA}$

(c) $\overline{BA}$

(d) $\{A\}$

(c) $\overline{AB}$

(d) $\{A\}$

⑥ Which of the following is true?

(a) $(AB)^2 = BD \times DC$

(b) $(AB)^2 = BC \times AC$

(c) $(AB)^2 = BD \times BC$

(d) $(AB)^2 = BC \times CD$

► Spot the mistake:

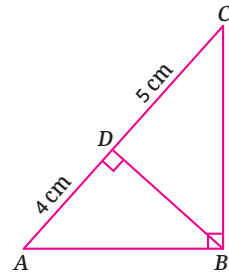
- ② In the given figure, what is the length of $\overline{CB}$?
Which of Magdy and Ibrahim has the correct answer? Discuss

Magdy's Solution:

$$\begin{aligned}(CB)^2 &= CD \times AD \\ &= 5 \times 4 = 20 \\ \therefore CB &= \sqrt{20} = 2\sqrt{5}\end{aligned}$$

Ibrahim's Solution:

$$\begin{aligned}(CB)^2 &= CD \times CA \\ &= 5 \times 9 = 45 \\ \therefore CB &= \sqrt{45} = 3\sqrt{5}\end{aligned}$$



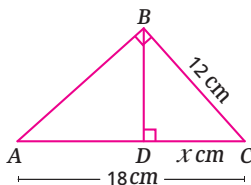
Second

Applying Concepts

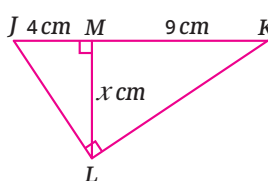


► In each of the following figures find the value of x

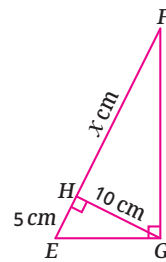
③



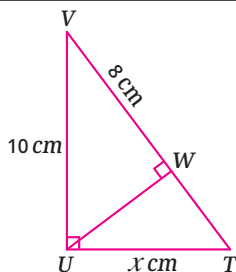
④



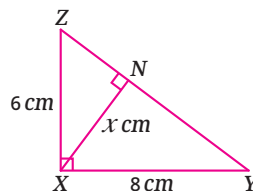
⑤



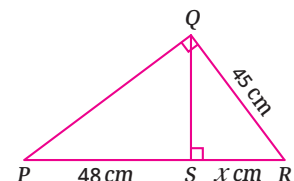
⑥



⑦



⑧

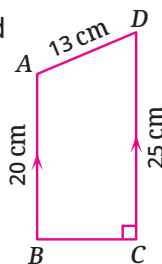


► In each of the following figures find the value of x

- ⑨ In the given figure:

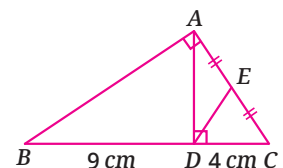
$ABCD$ is a right-angled trapezoid

- ① What is the length of the projection of $\overline{AD}$ on $\overrightarrow{DC}$?
- ② What is the length of the projection of $\overline{AD}$ on $\overrightarrow{BC}$?

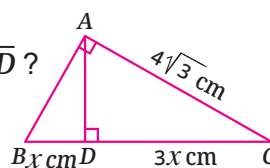


- ⑩ In the given figure:

What is the length of $\overline{DE}$?

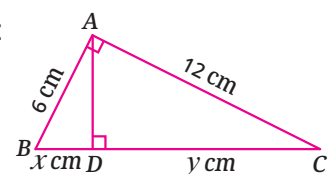


- ⑪ In the given figure:
What is the length of $\overline{BD}$?

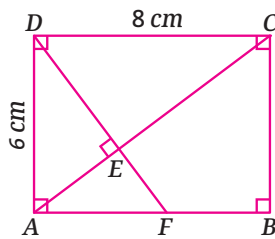


- ⑫ In the given figure:

What is the value of $\frac{x}{y}$?



- 13 In the given figure:
What is the length of $\overline{EF}$?



- 15 In the given figure:

ΔABC is right-angled at B ,

D is the midpoint of $\overline{BC}$,

$E \in \overline{AB}$ such that $m(\angle ADE) = 90^\circ$

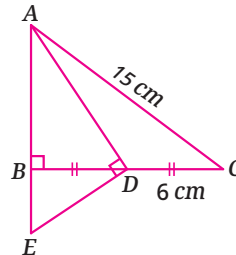
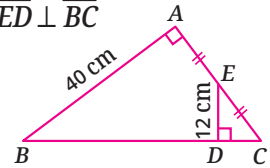
Find the length of $\overline{BE}$

- 14 In the given figure:

ΔABC is right-angled at A ,

E is the midpoint of $\overline{AC}$, $\overline{ED} \perp \overline{BC}$

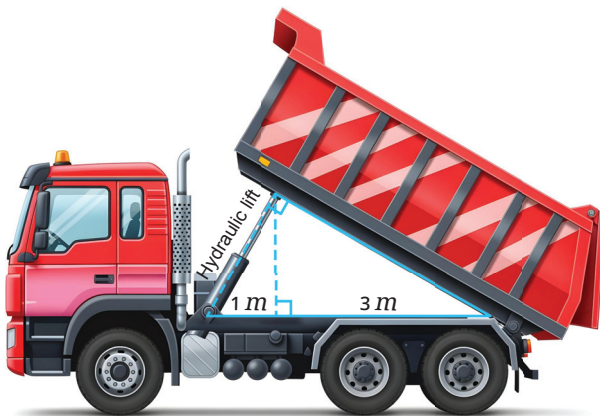
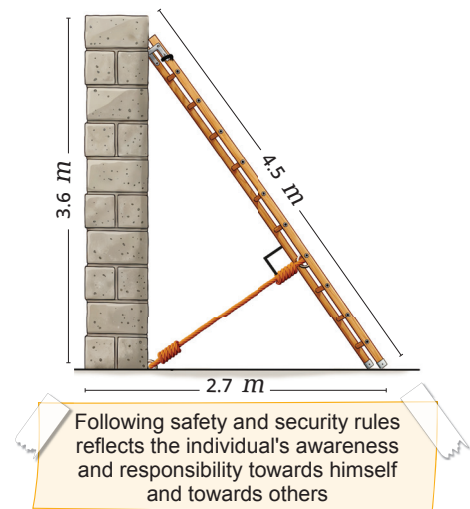
Find the length of $\overline{DC}$



Third Analysis and Subjects Integration



- 16 A ladder of length 4.5 meters, leans on a vertical wall, and its base is at a distance of 2.7 meters from the wall, and the height of its top from the ground is 3.6 meters. The ladder was tied with a rope perpendicular to the ladder as shown in the figure to prevent it from sliding, what is the length of the rope?



- 17 What is the length of the hydraulic lift in the position shown for the car?

Creative Thinking

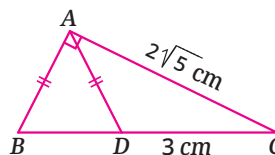


- 18 In the given figure:

ΔABC is right-angled at A ,

$D \in \overline{BC}$ where $AB = AD$,

Find the length of $\overline{BD}$



Evaluate your understanding!

How well do you understand the perpendicular projections and Euclid's theorem?

Tick the right box



Lesson 3 - 4

The Circle

■ Learning Outcomes

- Understand the concept of the circle.
- Understand the concept of diameter and radius.
- Use the compass to draw a circle given its center and radius length.
- Know the mathematical formula to calculate both circumference and area of the circle.
- Solve problems involving circumference and area of the circle.
- Use the mathematical formula to calculate both circumference and area of the circle in solving real life problems.

■ Vocabulary

- The circle
- Center of the circle
- Radius.
- Diameter
- Circumference
- Area of the circle

■ Note that

Usually, the circle is named of its center, so we say circle M to mean the circle whose center is the point M .

Get Ready!



The ferris wheel is one of the most prominent rides in amusement parks. It is a group of seats moving in a circular path, and every seat is at a fixed distance from its center. Can you calculate the distance covered by one of the seats if it rotates a full cycle around the center of the wheel, assuming that each seat is at a distance of 7 m from the center?

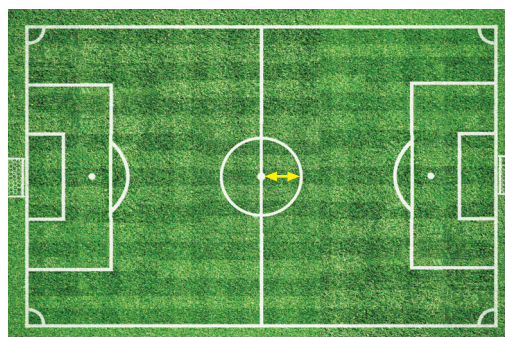


In this lesson, you will learn the concept of the circle and the concepts related to it, and how to determine its circumference and area of the circle; which will enable you to solve such real life problems.

Think & Discuss!



A football match begins with a kickoff from a point located at the center of the midfield circle, which is a circle located exactly in the middle of the field with a radius length of about 10 yards. How can this circle be drawn accurately on the field floor? Discuss.



Learn!



The circle

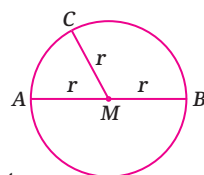
The circle is the set of points of the plane that are at a fixed distance from a fixed point in the plane.

- The fixed point is called "The center of the circle", and denoted by one of the Capital letters like « M », The fixed distance is called "The radius length" and is denoted by « r ».
- The radius is a line segment drawn between the center of the circle and any point on it

For Example: Each of $\overline{MC}$, $\overline{MB}$, $\overline{MA}$ is a radius in the circle M .

- The diameter is a line segment passing through the center of the circle and drawn between any two points on the circle.

For Example: $\overline{AB}$ is a diameter in the circle M and the diameter length is denoted by « d » and So $d = 2r$.



Drawing the circle

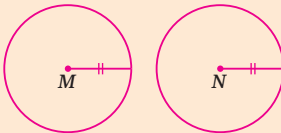
The ruler and compass are used to draw the circle given its radius length.

■ Self-Evaluation ①

Draw a circle N with a diameter length of 5 cm.

■ Note that

Congruent circles are circles that have equal radii.



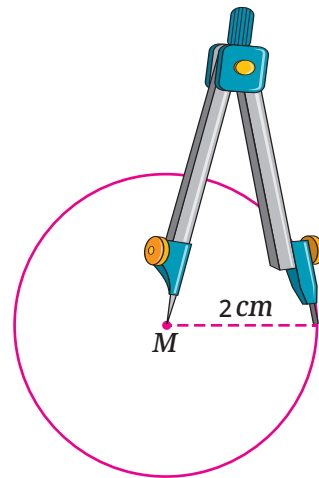
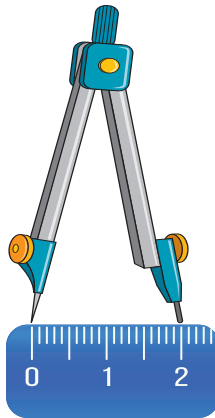
Example 1

Draw a circle M whose radius length is 2 cm.



To draw the circle M whose radius length is 2 cm, follow the following:

- ① Open the compass to a length equal to the required radius length (2 cm).
- ② Fix the compass point on the paper, so this point is the center of the circle M , then draw the circle. Now you get the required circle.



Practical Activity 1



Finding the ratio between the circumference and its diameter length

Use many circular objects such as a plate, wall clock, coin, compact disc (CD) and do the following:

- ① Cut a part of a thread with the length of the circumference of each object and use a ruler to measure the length of this part to the nearest millimeter.
- ② Measure the diameter length of each object to the nearest millimeter.
- ③ Record your measurements in the following table:

Object	Circumference (C)	Diameter length (d)	$\frac{C}{d}$
Plate	_____	_____	_____
Wall Clock	_____	_____	_____
Coin	_____	_____	_____
CD	_____	_____	_____

- ④ Use the calculator and find each time the ratio between the circumference to diameter length, and round the results to the nearest hundredth and record the results each time in the last column of the previous table. What do you notice about this ratio?



■ Note that

The number π is an irrational number where:

$$\pi \approx 3.141592654$$

and this value is an approximate value for the number π , for ease of calculations, we can use the following approximate values for the number π :

$$\pi \approx \frac{22}{7} \text{ or } \pi \approx 3.14$$

And these approximate values are the most common values.

■ Self-Evaluation ②

- ① Find the circumference of a circle with a diameter length of 25 feet ($\pi \approx 3.14$).
- ② Find in terms of π the circumference of a circle with a radius length of 12 cm.

■ Self-Evaluation ③

The diameter length of a car wheel is 70 cm, what is the circumference of the wheel?

If the wheel makes one complete revolution, how far does the car travel, how many revolutions does the wheel need to cover a distance of 2.2 km?

$$\left(\pi \approx \frac{22}{7}\right)$$



Circumference

- The circumference of a circle (C) is the length of the curved line representing the circle.
- The ratio between the circumference (C) and its diameter length (d) is an irrational number denoted by π .

Meaning that: $\frac{C}{d} = \pi$

Hence: $C = \pi d$ so $C = 2 \pi r$

Example ②

- ① Find the circumference of a circle with a diameter length of 32 cm, round the result to the nearest tenth ($\pi \approx 3.14$).
- ② Find in terms of π , the circumference of a circle with a radius length 13.5 feet.



$$\begin{aligned} \text{① } \because C &= \pi d \\ \therefore C &= 32 \pi \\ \therefore C &\approx 32 \times 3.14 \\ \therefore \text{The circumference} \\ &\approx 100.5 \text{ cm} \end{aligned}$$



$$\begin{aligned} \text{② } \because C &= 2 \pi r \\ \therefore C &= 2 \times 13.5 \pi \\ \therefore C &= 27 \pi \\ \therefore \text{The circumference} \\ &= 27 \pi \text{ feet} \end{aligned}$$

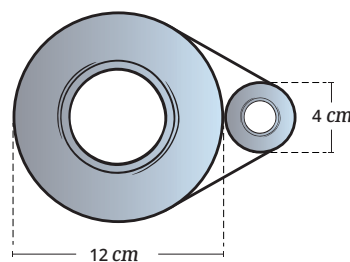
Note

Using the π key in the calculator gives more accurate results, because it uses a value for π rounded to the nearest billionth. Try yourself finding the circumference in part ① of the previous example using the π key in the calculator, and compare the results.

Example ③

Link to Life:

Find the number of revolutions that the small pulley rotates when the large pulley rotates 8 full revolutions.



$$\begin{aligned} \because \text{The circumference of the large pulley} &= \pi d = 12 \pi \text{ cm,} \\ \text{The circumference of the small pulley} &= \pi d = 4 \pi \text{ cm} \end{aligned}$$

$$\therefore \text{The number of revolutions that the small pulley rotates when the large pulley rotates one revolutions equals } \frac{12 \pi}{4 \pi} = 3 \text{ revolutions.}$$

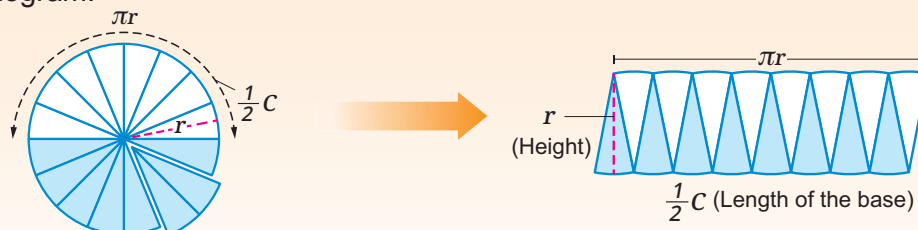
$$\therefore \text{The number of revolutions that the small pulley rotates when the large pulley rotates 8 revolutions equals } 8 \times 3 = 24 \text{ revolutions.}$$

Practical Activity 2



Deducing the mathematical formula to calculate The area of a circle

- 1 Fold a piece of paper in the shape of a circle into two halves for four consecutive times, to divide it into 16 equal parts.
- 2 Cut each part, then reassemble and arrange the parts to form a new shape closer to a parallelogram.



- 3 The height of the parallelogram is the radius length of the original circle (r), and its base length is half the circumference of the original circle ($\frac{1}{2} C = \pi r$).
- 4 The area of the circle = The area of the parallelogram = $\pi r \times r = \pi r^2$ meaning that $A = \pi r^2$.

Self-Evaluation 4

- 1 Find, in terms of π , the area of the circle whose radius length is 5 cm.
- 2 Using the π key in the calculator, find the area of the circle whose diameter length is 19 inches and round the result to the nearest tenth

Self-Evaluation 5

A circle whose circumference is 20π cm, find its area ($\pi \approx 3.14$).

Area of the circle

The area of the circle equals the product of the square of the radius length by the approximate ratio π

Meaning that $A = \pi r^2$.

Example 4

- 1 Using the π key in the calculator, find the area of the circle whose radius length is 9.2 cm, and round the result to the nearest tenth.
- 2 Find, in terms of π , the area of the circle whose diameter length is 8 feet.

✓ ① $\because A = \pi r^2$
 $\therefore A = (9.2)^2 \pi \approx 265.9$
 $\therefore$ The area of the circle $\approx 265.9 \text{ cm}^2$

② $\because d = 8 \quad \therefore r = \frac{8}{2} = 4$
 $\therefore A = \pi r^2$
 $\therefore A = (4)^2 \pi = 16\pi$
 $\therefore$ The area of the circle $= 16\pi$ square feet.

Example 5

A circle whose area is $81\pi \text{ cm}^2$, find its radius length, then find its circumference to the nearest tenth.

✓ $\because A = 81\pi \quad \therefore \pi r^2 = 81\pi$
 $\therefore r^2 = 81 \quad \therefore r = \sqrt{81} = 9$

Meaning that the radius length of the circle = 9 cm

$\therefore C = 2\pi r = 2 \times 9 \times \pi = 18\pi \approx 56.5$

Meaning that the circumference $\approx 56.5 \text{ cm}$

■ Self-Evaluation ⑥

A circular fountain with center M and radius length 4 m , it is required to make a circular path around it from tiles with a width of 2 m .

What is the area of the path in terms of π ?



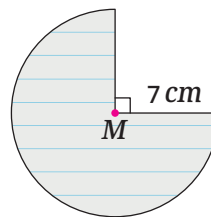
Example 6

Link to life :

A circular piece of paper from, which a quarter circle has been removed from it as in the figure.

Find the area and perimeter of the remaining part

$$(\pi \approx \frac{22}{7})$$



The area of the remaining part = $\frac{3}{4}$ The area of the circle

$$= \frac{3}{4} \pi r^2$$

$$= \frac{3}{4} \times \frac{22}{7} \times 7^2 = 115.5\text{ cm}^2$$

The perimeter of the remaining part = $\frac{3}{4}$ The circle circumference + $2r$

$$= 2r + 2\pi r \times \frac{3}{4}$$

$$= 2r + \frac{3}{2} \pi r$$

$$= 2 \times 7 + \frac{3}{2} \times \frac{22}{7} \times 7$$

$$= 14 + 33 = 47\text{ cm}$$

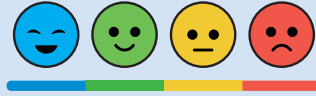
Collaborative Activity



Cooperate with your classmates to design a different set of circular-shaped medals, to gift them to the members of the winning team in the top students competition at your school. Calculate the circumference and area of each medal.



Lesson Assessment



First

Measuring Conceptual Understanding



► Choose the correct answer from the given ones:

- ① A circle whose radius length is 4 cm , what is its diameter length?
 (a) 2 cm (b) 4 cm
 (c) 8 cm (d) 16 cm
- ② In any circle, what is the value of $\frac{c}{d}$?
 (a) $\frac{\pi}{2}$ (b) $\frac{2}{\pi}$
 (c) π (d) 2π
- ③ What is the circumference of the circle whose diameter length is 12 cm ?
 (a) $3\pi\text{ cm}$ (b) $6\pi\text{ cm}$
 (c) $12\pi\text{ cm}$ (d) $36\pi\text{ cm}$
- ④ What is the area of the circle whose radius length is 10 m ?
 (a) $5\pi\text{ m}^2$ (b) $10\pi\text{ m}^2$
 (c) $25\pi\text{ m}^2$ (d) $100\pi\text{ m}^2$

Second

Applying Concepts

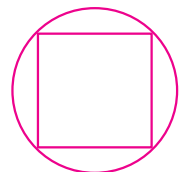


► Choose the correct answer from the given ones:

- ⑤ What is the radius length of a circle whose circumference is $13\pi\text{ cm}$?
 (a) $\frac{\pi}{2}\text{ cm}$ (b) 6.5 cm
 (c) 13 cm (d) 26 cm
- ⑥ What is the diameter length of a circle whose area is 36π square inches?
 (a) 6 inches (b) 12 inches
 (c) 18 inches (d) 36 inches
- ⑦ The areas of two circles are 64π , 100π , what is the ratio between their circumferences?
 (a) $3:4$ (b) $4:5$
 (c) $9:16$ (d) $16:25$
- ⑧ The circumference of a circle is $10\pi\text{ cm}$, what is its area?
 (a) $25\pi\text{ cm}^2$ (b) $50\pi\text{ cm}^2$
 (c) $75\pi\text{ cm}^2$ (d) $100\pi\text{ cm}^2$

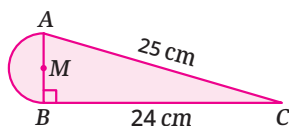
► Answer the following questions:

- ⑨ In each of the following draw a circle, then calculate its circumference and area:
 - ① If the radius length is 3 cm ($\pi \approx 3.14$)
 - ② If the diameter length is 7 cm ($\pi \approx \frac{22}{7}$)
- ⑩ The circumference of a circle is 18.84 cm . Find its radius length and its area ($\pi \approx 3.14$)?
- ⑪ The area of a circle is 78.5 square feet . Find its radius length and circumference ($\pi \approx 3.14$)?
- ⑫ In the given figure:
 If the circumference of the circle is 75.36 cm .
 Find the area of the square (knowing that $\pi \approx 3.14$)

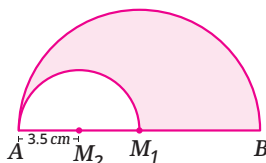


- 13 In each of the following, find perimeter and area of the colored part (knowing that $\pi \approx \frac{22}{7}$)

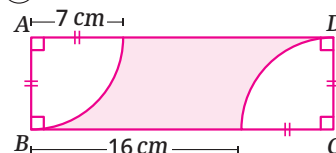
1



2



3



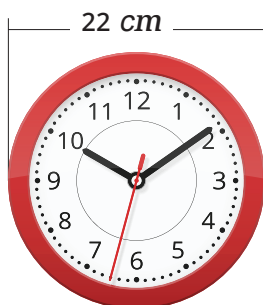
Third

Analysis and Subjects Integration



- 14 Calculate, to the nearest tenth, what is required below each figure:

1



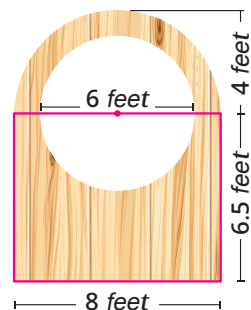
The area of the clock

2



The circumference of the coin

3



The area of the wood

- 15 **Decor** : Amal bought a circular rug with a diameter length of 2 m.
What is the area of the rug to the nearest square meter?
If the cost of a square meter of the rug is 150 pounds,
What is the total cost to the nearest pound?



Caring for decor reflects an aesthetic sense, and expresses high taste.



It is necessary to check the brakes and tires before driving the bicycle, and drive at an appropriate speed to maintain safety

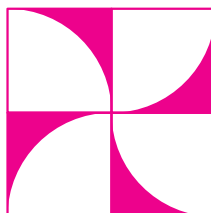
- 16 **Sport** : Rozan's bicycle contains two small balance wheels fixed in the rear wheel to prevent it from falling. If the front and rear wheels of the bicycle are congruent and the diameter length of each is 42 cm, and the diameter length of the balance wheel is 14 cm. If the bicycle covered a distance of 66 m, calculate the difference between the number of revolutions that the balance wheel rotates and the number of revolutions that the front wheel rotates respectively ($\pi \approx \frac{22}{7}$).

Creative Thinking



- 17 In the given figure:

If the side length of the square is 6 cm
Calculate the area of the colored region to the nearest tenth.



Evaluate your understanding!

How well do you understand The circle?
Tick the right box



Lesson 3 - 5

Right Circular Cylinder and Right Prism

■ Learning Outcomes

- Understand the concept of both (right circular cylinder and right prism).
- Calculate the volume of the cylinder, and its lateral area and total area.
- Calculate the volume of the prism, and its lateral area and total area.
- Use the laws to calculate volume, lateral area, and total area for both the cylinder and prism in solving life problems.

■ Vocabulary

- Prism
- Cylinder
- Volume
- Lateral area
- Surface area

Get Ready!



Wadi Degla Protectorate is a nature reserve located in Maadi region east of Cairo. It is distinguished by its ancient limestone rock formations, and various types of plants and wild animals live there such as deer and foxes, and it is a distinguished place for camping and nature exploration lovers.



Osama set up a tent in the shape of a right triangular prism made of cloth, can you calculate the total area of the cloth used in making the tent?

In this lesson, you will learn how to calculate the volume and total area for both the right prism and the right circular cylinder, which will enable you to solve such problems.

Think & Discuss!



A beverage shop uses cylindrical cups with a height of 10 cm, and a radius length of its base 3.5 cm. If the shop owner wanted to print a logo on the lateral surface of each cup, how much is the available area for printing for one cup? Discuss.



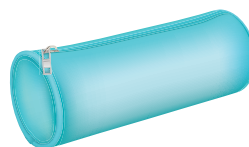
Learn!



First Right Circular Cylinder

- The right circular cylinder is a three-dimensional solid, having two congruent and parallel circular bases, and its lateral surface is perpendicular to the two bases.
- The height of the right circular cylinder is the length of the perpendicular drawn between the centers of its two bases.

Examples for the right circular cylinder :



■ Note that

When unfolding the right circular cylinder we get a rectangular surface in addition to the surfaces of two circles which are the bases of the cylinder.

■ Critical Thinking

What happens to the volume of the cylinder if :

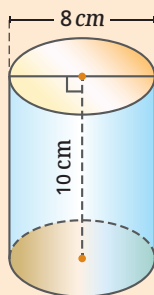
- ① Its height doubled?
- ② Its base radius length doubled?
- ③ Both its height and its base radius length doubled?

■ Note that

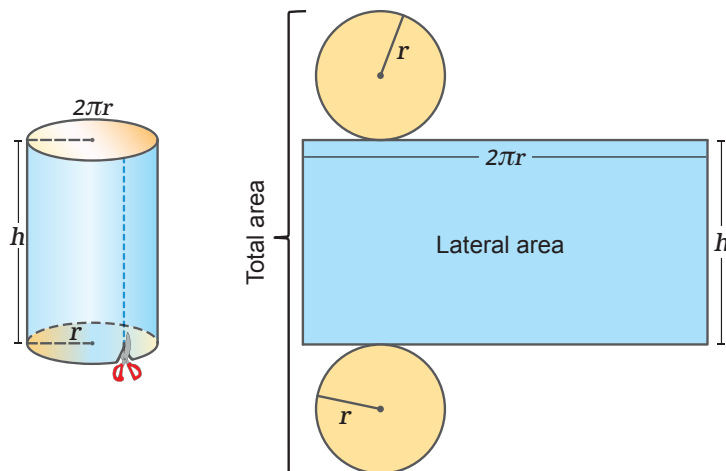
The total area sometimes is called the surface area.

■ Self-Evaluation ①

In the following figure :
Find the volume, lateral area, and surface area of the cylinder in terms of π .



Volume of the right circular cylinder and its lateral area and total area



If the length of the circular base radius = r and the height of the cylinder = h then:

The base circumference = $2\pi r$ and the base area = πr^2 and hence :

- The volume of the cylinder (V) = base area $\times$ height

Meaning that : $V = \pi r^2 h$

- The lateral area of the cylinder ($L.A$) = base circumference $\times$ height

Meaning that : $L.A = 2\pi r h$

- The total area of the cylinder ($S.A$)

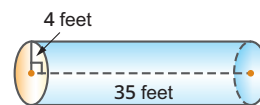
= lateral area of the cylinder + double the base area

Meaning that : $S.A = 2\pi r h + 2\pi r^2$
 $= 2\pi r (h + r)$

Example ①

In the given figure:

Find the volume, lateral area, and surface area of the cylinder rounding the result to the nearest tenth.



$$\therefore r = 4, h = 35$$

$$\therefore V = \pi r^2 h = \pi (4)^2 (35) = 560\pi \approx 1759.3$$

$$\therefore \text{The volume of the cylinder} \approx 1759.3 \text{ cubic feet}$$

$$\therefore L.A = 2\pi r h = 2\pi (4) (35) = 280\pi \approx 879.6$$

$$\therefore \text{The lateral area} \approx 879.6 \text{ square feet}$$

$$\therefore S.A = 2\pi r (h + r) = 2\pi (4) (35 + 4) = 312\pi \approx 980.2$$

$$\therefore \text{The total area (surface area)} \approx 980.2 \text{ square feet}$$

■ Self-Evaluation ②

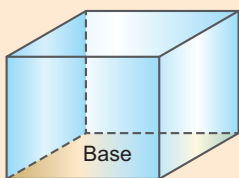
A right circular cylinder whose lateral area is 110 m^2 , and its base diameter length is 10 m .

Find its volume and its total area

(Where $\pi \approx \frac{22}{7}$)

■ Note that

If all faces of the right prism are rectangles as in the rectangular prism (cuboid).



It is necessary to determine which of these rectangles is considered a base for this prism.

■ Previous Knowledge

- Perimeter of any shape = sum of its side lengths
- Area of triangle = $\frac{1}{2}$ base length $\times$ height
- Area of rectangle = length $\times$ width
- Area of square = side length $\times$ itself
- Area of trapezium = half sum of the two parallel base lengths $\times$ height

Example 2

A right circular cylinder whose volume is 125.6 cm^3 and its height is 2.5 cm . Find its total area ($\pi \approx 3.14$)



$\therefore$ The volume of the cylinder = 125.6 cm^3

$$\therefore V = \pi r^2 h$$

$$\therefore 125.6 = 3.14 (r^2) (2.5)$$

$$\therefore r^2 = 16$$

$$\therefore r = \sqrt{16} = 4$$

$$\therefore S.A = 2\pi r(h + r) = 2(3.14)(4)(2.5 + 4) = 163.28$$

$\therefore$ Total area of the cylinder equals 163.28 cm^2

Second the right prism

- The right prism is a three-dimensional solid having two congruent and parallel bases each of them is a polygon and its lateral edges are perpendicular to its two bases.
- All lateral faces of the right prism are rectangles.
- The height of the right prism is the length of any lateral edge.

Examples for the Prism :

The prism is named according to the shape of its base as follows :



Base is a triangle
"Triangular prism"

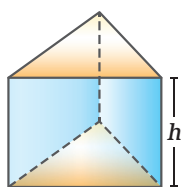


Base is a quadrilateral
"Quadrilateral prism"

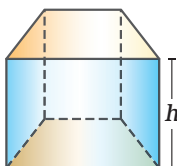
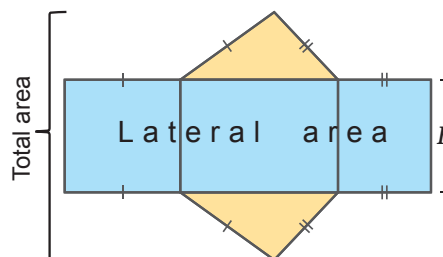


Base is a pentagon
"Pentagonal prism"

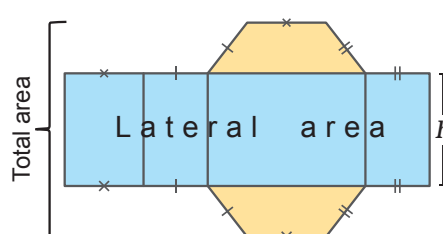
Volume of the right prism and its lateral area and total area

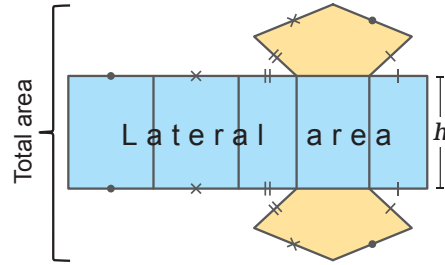
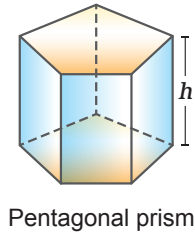


Triangular prism



Quadrilateral prism

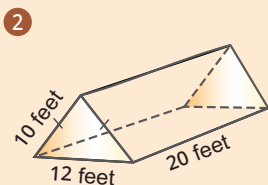
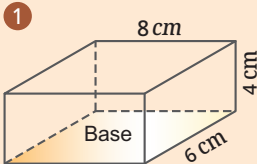




- The volume of the right prism (V) = base area $\times$ height
- The lateral area of the right prism ($L.A$) = base perimeter $\times$ height
- The total area of the right prism ($S.A$)
= lateral area + double the base area.

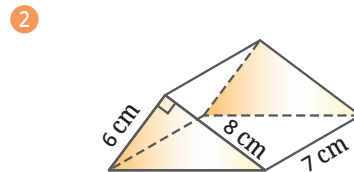
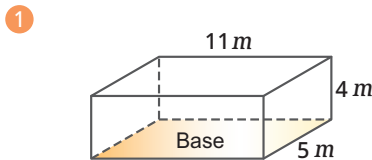
■ Self-Evaluation ③

Find the surface area and volume for each of the two following solids:



Example 3

Find the total area and volume for each of the two following solids:

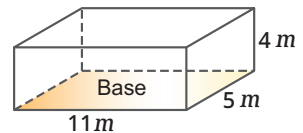


✓ ① Base area = $11 \times 5 = 55 \text{ m}^2$

• The lateral area = $2(5 + 11) \times 4 = 128 \text{ m}^2$

• The total area = $128 + 2 \times 55 = 238 \text{ m}^2$

• The volume = $55 \times 4 = 220 \text{ m}^3$



② We notice that the base is the triangular face.

From Pythagoras theorem you can find that:

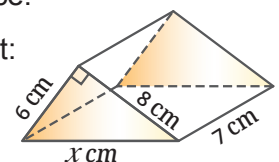
$$x = \sqrt{6^2 + 8^2} = 10$$

• The base area = $\frac{1}{2} \times 6 \times 8 = 24 \text{ cm}^2$

• The lateral area = $(6 + 8 + 10) \times 7 = 168 \text{ cm}^2$

• The total area = $168 + 2 \times 24 = 216 \text{ cm}^2$

• The volume = $24 \times 7 = 168 \text{ cm}^3$



■ Self-Evaluation ④

A quadrilateral prism whose base is in the shape of a square with side length 4 cm and its volume is 192 cm^3 . Find its surface area.

■ Self-Evaluation ⑤

A cube of metal, the sum of its edge lengths is 144 cm . It was melted and made from its material 16 congruent prisms the base of each is in the shape of a square with side length 6 cm . Find the height of each Prism.

Example 4

A quadrilateral prism whose base is in the shape of a rectangle with dimensions 8 cm , 4 cm and its total area is 280 cm^2 , find its volume.

$$\textcircled{1} \because \text{Base area} = 8 \times 4 = 32\text{ cm}^2,$$

$$\text{Base perimeter} = 2(4 + 8) = 24\text{ cm}$$

$$\because \text{The total area} = 280\text{ cm}^2$$

$$\therefore 24 \times h + 32 \times 2 = 280 \quad \therefore h = 9$$

$$\therefore \text{The volume of the prism} = 32 \times 9 = 288\text{ cm}^3$$

Example 5

An alloy in the shape of a triangular prism whose base is a right-angled triangle, the lengths of the two right-angle sides are 9 cm , 12 cm and the height of the prism is 20 cm . It was melted and transformed into number of cubes each with edge length 3 cm . What is the number of these cubes ?

$$\begin{aligned} \checkmark \because \text{The volume of the triangular prism} &= \text{base area} \times \text{height} \\ &= \frac{1}{2} \times 9 \times 12 \times 20 \\ &= 1,080\text{ cm}^3 \end{aligned}$$

$$\text{The volume of the cube} = 3 \times 3 \times 3 = 27\text{ cm}^3$$

$$\text{The number of cubes} = \frac{1,080}{27} = 40 \text{ cubes.}$$

Lesson Assessment



First

Measuring Conceptual Understanding



► Choose the correct answer from the given ones:

- ① A right circular cylinder whose base diameter length is 10 cm , and its height is 15 cm , what is its volume?
 (a) $75\pi\text{ cm}^3$ (b) $150\pi\text{ cm}^3$
 (c) $375\pi\text{ cm}^3$ (d) $1500\pi\text{ cm}^3$
- ② A right circular cylinder whose base radius length is 5 cm , and its height is 15 cm , what is its surface area?
 (a) $75\pi\text{ cm}^2$ (b) $200\pi\text{ cm}^2$
 (c) $250\pi\text{ cm}^2$ (d) $375\pi\text{ cm}^2$
- ③ A right quadrilateral prism whose base is in the shape of a square with side length 10 cm . If the height of the prism is 8 cm , What is its volume?
 (a) 80 cm^3 (b) 160 cm^3
 (c) 320 cm^3 (d) 800 cm^3
- ④ If the lateral area of a right circular cylinder equals $2\pi r^2\text{ cm}^2$, what is its height?
 (a) $\frac{1}{2}r\text{ cm}$ (b) $\frac{3}{4}r\text{ cm}$
 (c) $r\text{ cm}$ (d) $2r\text{ cm}$
- ⑤ A right pentagonal prism whose base is a regular pentagon with area 10 m^2 and the area of one of its lateral rectangles is 9 m^2 , what is the total area of the prism?
 (a) 19 m^2 (b) 38 m^2 (c) 55 m^2 (d) 65 m^2

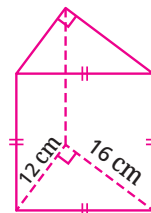
Second

Applying Concepts

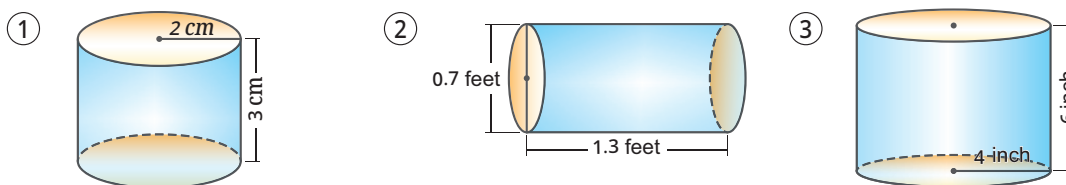


► Choose the correct answer from the given ones:

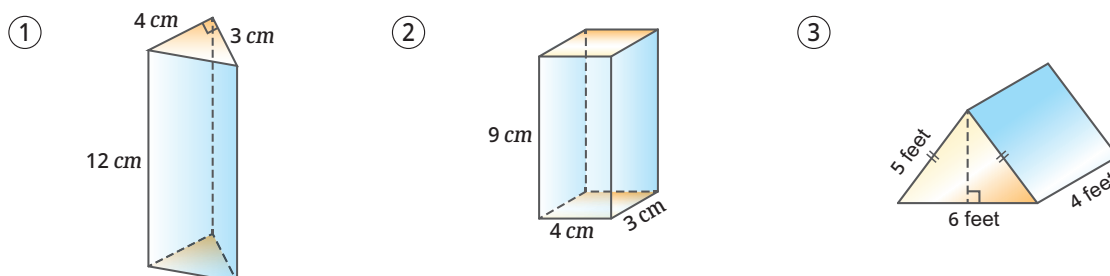
- ⑥ A right circular cylinder whose height is 6 inches and its base radius length equals half its height, what is its volume in cubic inches?
 (a) 18π (b) 36π
 (c) 54π (d) 108π
- ⑦ A right circular cylinder whose volume is $320\pi\text{ cm}^3$, and its height is 20 cm , what is the diameter length of its base?
 (a) 2 cm (b) 4 cm
 (c) 8 cm (d) 16 cm
- ⑧ If the base of a right prism is an equilateral triangle with side length 5 cm , and the height of the prism equals 4 cm , what is the lateral area of the prism?
 (a) 15 cm^2 (b) 30 cm^2
 (c) 60 cm^2 (d) 240 cm^2
- ⑨ A right pentagonal prism whose height is 8 cm , and its volume is 168 cm^3 , what is its base area?
 (a) 1344 cm^2 (b) 176 cm^2
 (c) 21 cm^2 (d) 18 cm^2
- ⑩ A right circular cylinder whose base radius length equals its height, if the volume of the cylinder is $125\pi\text{ cm}^3$, what is its lateral area?
 (a) $150\pi\text{ cm}^2$ (b) $125\pi\text{ cm}^2$
 (c) $100\pi\text{ cm}^2$ (d) $50\pi\text{ cm}^2$
- ⑪ In the given figure :
 What is the volume of the triangular prism ?
 (a) 1960 cm^3
 (b) 1920 cm^3
 (c) 1260 cm^3
 (d) 960 cm^3



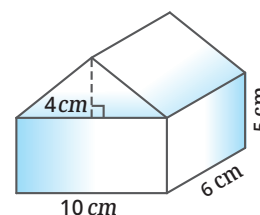
- 12 Find the volume and the surface area for each of the following solids ($\pi \approx 3.14$):



- 13 Find the volume and the surface area for each of the following solids:



- 14 Find the volume of the pentagonal prism shown in the figure:



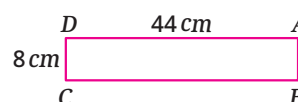
- 15 A right circular cylinder whose volume is $45\pi \text{ cm}^3$ and its height is 5 cm and its base radius length equals the edge length of a cube. Find the surface area of the cube.
- 16 The base of a triangular prism is a right-angled triangle, its hypotenuse length is 17 meters, and one of its side lengths is 8 meters. If the height of the prism is 12 meters, what is its volume?
- 17 A right circular cylinder whose base radius length is 12 cm and its height is 22 cm, if its base radius length increased by 25% and its height remained constant, calculate the volume of the cylinder after the increase.
- 18 A right pentagonal prism of metal, its base area is 105 cm^2 and its height is 11 cm. It was melted and transformed into number of congruent right circular cylinders, the height of each is 2 cm and its base radius length is 3.5 cm. Find the number of these cylinders ($\pi \approx \frac{22}{7}$).

Third

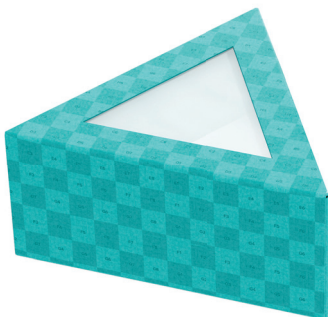
Analysis and Subjects Integration



- 19 The given figure represents a piece of paper in the shape of a rectangle $ABCD$, It is folded in the shape of a right circular cylinder such that $\overline{AB}$ coincides with $\overline{DC}$. Find the volume of the resulting cylinder ($\pi \approx \frac{22}{7}$).



- ②0 Cylindrical tanker. Its base radius length is 7 feet. Find the volume of the tank in cubic feet.

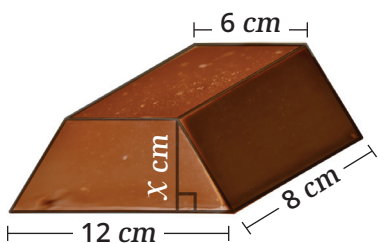


- ②1 A gift box in the shape of a triangular prism. Its base is an equilateral triangle with side length 20 cm and the height of the prism is 10 cm. Calculate both the lateral area and volume of the box.

- ②2 A cylindrical water tank, its base radius length is 0.5 meters and its height is 1.2 meters. If Magdy wanted to paint its lateral surface with paint, the price of square meter of this paint is 210 pounds. Calculate the cost of the paint.



Water is the basis of life. Rationalizing water consumption is evidence of human awareness and civilization.



- ②3 A piece of chocolate in the shape of a quadrilateral prism, if its volume is 360 cm^3 , find the value of X .

Creative Thinking



- ②4 Choose the correct answer from the given ones:
If the height of a right circular cylinder equals four times the circumference of the cylinder base (C), what is the volume of the cylinder in terms of (C)?

(a) $\frac{C^3}{\pi}$ (b) $\frac{2C^3}{\pi}$ (c) $\frac{2C^2}{\pi^2}$ (d) $4\pi C^3$

Evaluate your understanding!

How well do you understand the right circular cylinder and the right prism? Tick the right box



Lesson 3 - 6

The Distance between Two Points in The Coordinate Plane

■ Learning Outcomes

- Understand the mathematical formula to calculate the distance between two points in the coordinate plane.
- Use the mathematical formula to calculate the distance between two points to solve problems.

■ Vocabulary

- The distance between two points
- The coordinate plane

■ Note that

The square of any quantity is always non-negative; therefore when squaring the expression it is not necessary to take the absolute value.

■ Discuss

The law can be written in the formula:

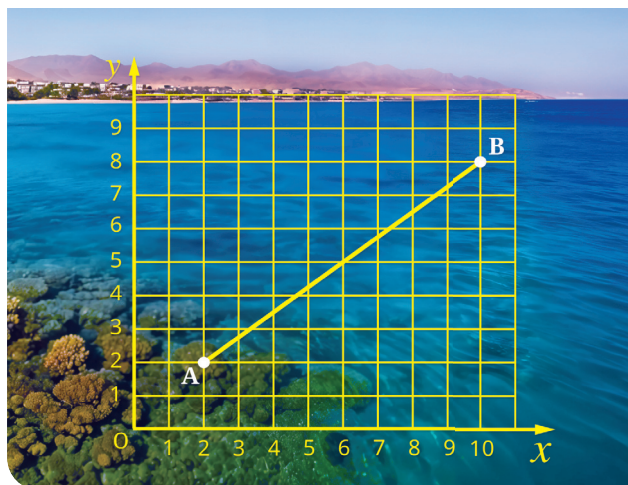
$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Explain that.

Get Ready!



Sharm El-Sheikh is one of the most beautiful tourist cities in Egypt, and is famous for its wonderful beaches and colorful coral reefs that attract divers from all over the world. A team of marine environmental scientists works on studying the distances between different diving sites in the Red Sea to determine coral reef distribution areas. Can you help them to determine the distance between the two sites A , B in the picture?

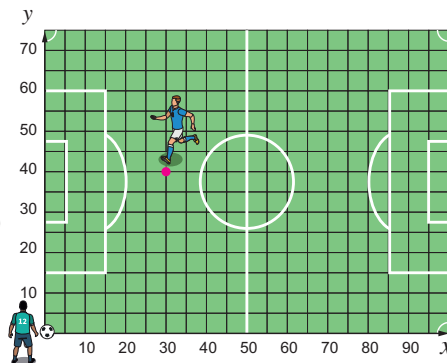


In this lesson, you will learn the mathematical formula to calculate the distance between two points in the coordinate plane; which will enable you to solve such real life problems.

Think & Discuss!



In a football match, Magdy wants to pass the ball from the corner of the field at the point $(0, 0)$ to his friend Osama standing at the point $(30, 40)$. Can you know the distance between Magdy and Osama? Discuss.



Learn!



The distance between two points in the coordinate plane

The distance " d " between the two points $A(x_1, y_1)$, $B(x_2, y_2)$ is given by the mathematical formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

You can deduce the previous law as follows:

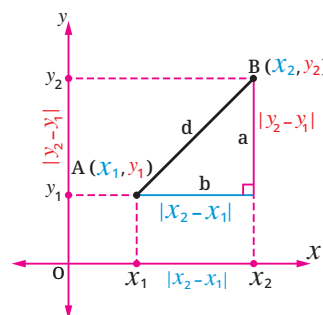
Assuming that $A(x_1, y_1)$, $B(x_2, y_2)$
And the distance between them is d , then:

$$a = |y_2 - y_1|, \quad b = |x_2 - x_1|$$

And from Pythagoras theorem you find that:

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\therefore d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



■ Self-Evaluation ①

- ① Find the length of $\overline{AB}$ if $A(5, -7)$, $B(2, -3)$
- ② If the distance between two points $A(-1, 3)$, $B(k, 5)$ equals $2\sqrt{5}$ units, find the possible values of k

■ Self-Evaluation ②

Find the perimeter of $\triangle ABC$ in which $A(5, 6)$, $B(29, 13)$, $C(5, 31)$

Example 1

- ① Find the distance between the two points:
 $A(6, -3)$, $B(-2, 3)$



①

$$\begin{aligned}\therefore d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ \therefore d &= \sqrt{(-2 - 6)^2 + (3 - (-3))^2} \\ &= \sqrt{(-8)^2 + (6)^2} \\ &= \sqrt{64 + 36} = \sqrt{100} = 10\end{aligned}$$

$\therefore$ The distance between A , B is 10 units.

- ② If the distance between the two points $A(4, k)$, $B(1, 7)$ equals 5 units, find the possible values of k

$$\begin{aligned}\textcircled{2} \therefore d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ \therefore \sqrt{(1 - 4)^2 + (7 - k)^2} &= 5 \\ \therefore (-3)^2 + (7 - k)^2 &= (5)^2 \\ \therefore 9 + (7 - k)^2 &= 25 \\ \therefore (7 - k)^2 &= 25 - 9 = 16 \\ \therefore 7 - k &= \pm \sqrt{16} \\ \therefore 7 - k &= 4 \quad , \quad 7 - k = -4 \\ \therefore k &= 7 - 4 \quad \quad k = 7 + 4 \\ \therefore k &= 3 \quad \quad k = 11\end{aligned}$$

Example 2

Find the perimeter of $\triangle ABC$ in which $A(-1, 2)$, $B(13, 2)$, $C(4, 14)$

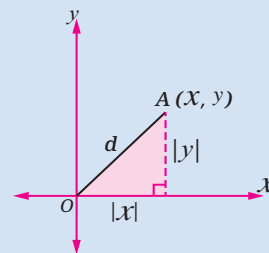


$$\begin{aligned}\therefore d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ \therefore AB &= \sqrt{(13 - (-1))^2 + (2 - 2)^2} = \sqrt{196} = 14 \\ AC &= \sqrt{(4 - (-1))^2 + (14 - 2)^2} = \sqrt{169} = 13 \\ BC &= \sqrt{(4 - 13)^2 + (14 - 2)^2} = \sqrt{225} = 15 \\ \therefore \text{Perimeter of } \triangle ABC &= 14 + 13 + 15 = 42 \text{ units.}\end{aligned}$$

Distance between a point and the origin point

The distance " d " between the point $A(x, y)$ and the origin point $O(0, 0)$ is given by the mathematical formula:

$$d = \sqrt{x^2 + y^2}$$



For example:

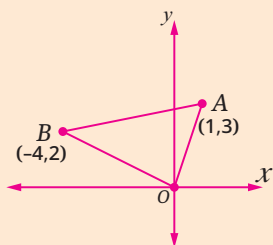
If $A(-3, 4)$, d is the distance between the point A and the origin point, then:

$$d = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + (4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

Meaning that $A(-3, 4)$ is at a distance of 5 units from the origin point.

■ Self-Evaluation ③

In the following figure:



Calculate the perimeter of $\triangle OAB$ rounding the result to the nearest tenth of unit.

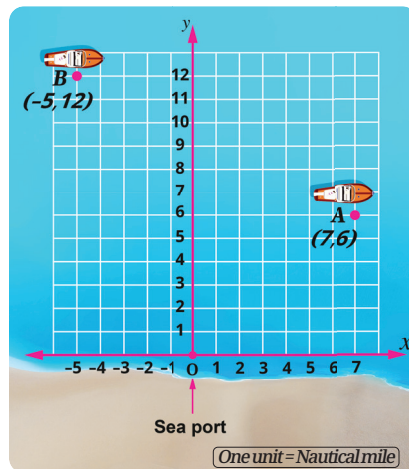
■ Self-Evaluation ④

Using the law of the distance between two points, which of the following sets of points are collinear:

- ① $A(4, 7)$
 $B(-5, 19)$
 $C(7, 3)$
- ② $A(5, -5)$
 $B(-1, 7)$
 $C(15, 15)$.

Example 3

The given figure shows the positions of two ships A , B relative to a sea port O , find the distance of each ship from the sea port. Which of the two ships is closer to the port?



✓ $\therefore d = \sqrt{x^2 + y^2}$

For the ship in position A :

$$d = \sqrt{7^2 + 6^2} = \sqrt{49 + 36} = \sqrt{85} \approx 9.2$$

$\therefore$ Ship A is at a distance of 9.2 nautical miles approximately from the port.

For the ship in position B :

$$d = \sqrt{(-5)^2 + (12)^2} = \sqrt{25 + 144} = \sqrt{169} = 13$$

$\therefore$ Ship B is at a distance of 13 nautical miles from the port.

$\therefore$ Ship A is closer to the port than Ship B .

Note that:

• For any three points A , B , C , if $\overline{AC}$ is longer than each of $\overline{AB}$, $\overline{BC}$ then:

- ① The points A , B , C are collinear, if $AB + BC = AC$.
- ② The points A , B , C are vertices of a triangle, if $AB + BC > AC$.

Example 4

Using the law of the distance between two points, prove that the points $A(-2, 3)$, $B(0, 4)$, $C(4, 6)$ are collinear.

✓ $\therefore d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

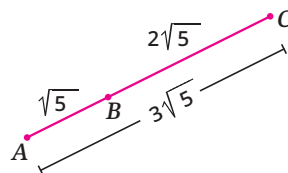
$$\therefore AB = \sqrt{(0 + 2)^2 + (4 - 3)^2} = \sqrt{4 + 1} = \sqrt{5}$$

$$AC = \sqrt{(4 + 2)^2 + (6 - 3)^2} = \sqrt{36 + 9} = 3\sqrt{5}$$

$$BC = \sqrt{(4 - 0)^2 + (6 - 4)^2} = \sqrt{16 + 4} = 2\sqrt{5}$$

$$\therefore AC = AB + BC$$

$\therefore$ The points A , B , C are collinear, $B \in \overline{AC}$



■ Self-Evaluation ⑤

Prove that the points
 $A(3, -2)$, $B(4, 1)$,
 $C(-2, 3)$, $D(-3, 0)$
 are vertices of a
 rectangle.

■ Self-Evaluation ⑥

If the point $A(15, -9)$
 is equidistant from the
 two points
 $B(-5, k)$, $C(8, 15)$
 Find the possible values
 of k .

Example 5

Prove that the points $A(1, -2)$, $B(2, 2)$, $C(-1, 2)$, $D(-2, -2)$ are vertices of a parallelogram.



$$\therefore d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\therefore AB = \sqrt{(1 - 2)^2 + (-2 - 2)^2} = \sqrt{17}$$

$$BC = \sqrt{(2 + 1)^2 + (2 - 2)^2} = \sqrt{9} = 3$$

$$CD = \sqrt{(-2 + 1)^2 + (-2 - 2)^2} = \sqrt{17}$$

$$AD = \sqrt{(-2 - 1)^2 + (-2 + 2)^2} = \sqrt{9} = 3$$

$$\therefore AB = CD, BC = AD$$

Meaning that every two opposite sides are equal in length.

$\therefore$ The figure $ABCD$ is a parallelogram.

Example 6

If the two points $X(5, k)$, $Y(-4, 6)$ lie on a circle whose center is $M(1, 2)$, find the possible values of k , and calculate the area of the circle M .



$$\therefore d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\therefore MY = \sqrt{(-4 - 1)^2 + (6 - 2)^2} = \sqrt{25 + 16} = \sqrt{41}$$

$\therefore$ The radius length of the circle M passing through the two points X , Y equals $\sqrt{41}$ unit length.

$$\therefore MX = MY = r \quad (\text{Length of two radii in the circle } M)$$

$$\therefore \sqrt{(5 - 1)^2 + (k - 2)^2} = \sqrt{41}$$

$$\therefore 16 + (k - 2)^2 = 41 \quad \therefore (k - 2)^2 = 25$$

$$\therefore k - 2 = \pm \sqrt{25}$$

$$\therefore k - 2 = 5 \quad \text{or} \quad k - 2 = -5$$

$$\therefore k = 5 + 2 \quad \quad \quad k = -5 + 2$$

$$\therefore k = 7 \quad \quad \quad k = -3$$

$$\therefore A = \pi r^2 \quad \quad \quad \therefore A = \pi \times (\sqrt{41})^2 = 41\pi$$

Meaning that the area of the circle M equals 41π square units.

Lesson Assessment



First

Measuring Conceptual Understanding



► Choose the correct answer from the given ones:

- ① What is the distance between the point $A(-3, 4)$ and the origin point?
 (a) 3 length units (b) 4 length units
 (c) 5 length units (d) 7 length units
- ② If $A(0, 4)$, $B(-3, 0)$ what is length of $\overline{AB}$?
 (a) 1 length units (b) 3 length units
 (c) 5 length units (d) 7 length units

► Spot the mistake:

- ③ Both Amal and Samar calculated the distance between the two points $A(-5, 3)$, $B(4, 15)$, and their answers were as follows:

Amal's Solution:

$$AB = \sqrt{(4+5)^2 + (15-3)^2}$$

$$AB = (4+5) + (15-3)$$

$$AB = 9 + 12 = 21$$

Samar's Solution:

$$AB = \sqrt{(-5-4)^2 + (3-15)^2}$$

$$AB = \sqrt{(-9)^2 + (-12)^2}$$

$$AB = \sqrt{81 + 144} = \sqrt{225}$$

$$AB = 15$$

Which of the two solutions is correct? Why?

Second

Applying Concepts



- ④ Find length of $\overline{AB}$ in each of the following:

① $A(-1, 2)$, $B(3, 5)$

② $A(2, 1)$, $B(-2, 3)$

③ $A(8, -4)$, $B(-7, 4)$

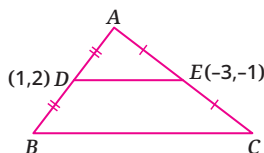
► Choose the correct answer from the given ones:

- ⑤ If the distance between the two points $A(X, 0)$, $B(3, 2)$ equals $2\sqrt{5}$ unit length, what is the set of possible values of X ?
 (a) $\{-1, 7\}$ (b) $\{7\}$ (c) $\{-4, 4\}$ (d) $\{-4, 7\}$
- ⑥ What is the area of the circle whose center is the point $A(-3, 5)$, and passes through the point $B(9, -4)$?
 (a) 15π square units (b) 225π square units
 (c) 13π square units (d) 144π square units

- ⑦ In the given figure:

What is the length of $\overline{BC}$?

- (a) 12 length units
 (b) 10 length units
 (c) 8 length units
 (d) 5 length units

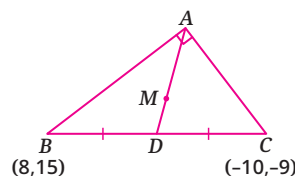


- ⑧ In the given figure:

If M is the intersection point of the medians of the triangle ABC ,

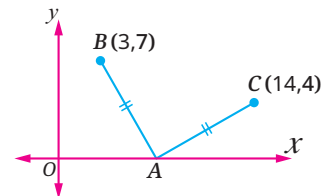
What is the length of $\overline{DM}$?

- (a) 5 length units (b) 6 length units
 (c) 7 length units (d) 8 length units



► Answer the following questions:

- 9 Prove that the points $A(-3, 7)$, $B(3, -2)$, $C(-1, 4)$ are collinear.
- 10 Prove that the points $A(7, -4)$, $B(-1, 2)$, $C(5, -6)$ are vertices of a triangle, and determine its type with respect to its angles.
- 11 Prove that the points $A(3, 2)$, $B(8, 7)$, $C(1, 4)$ are vertices of a right-angled triangle and calculate its area.
- 12 Prove that the points $A(2, 3)$, $B(1, -1)$, $C(-4, -3)$, $D(-3, 1)$ are vertices of a parallelogram using the law of distance between two points.
- 13 In the given figure:
Find the coordinates of point A

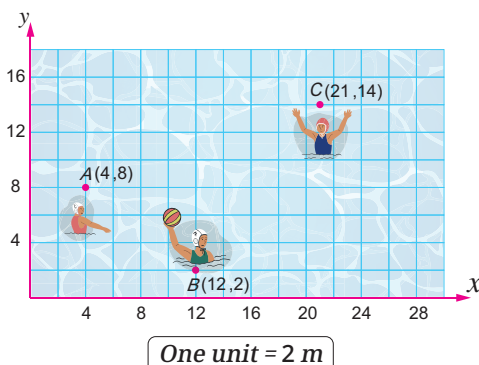
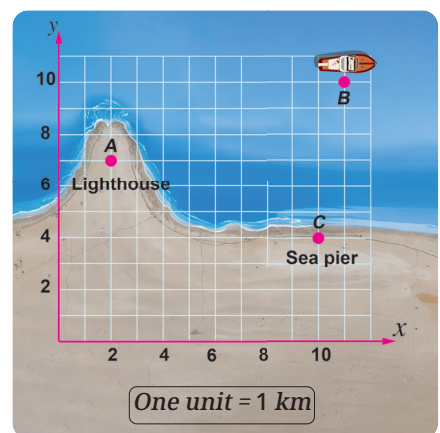


Third Analysis and Subjects Integration



14 **The Marine Safety**

The lighthouse casts light in a circular path so that the maximum range of light is that it reaches the sea pier, does the light reach the ship in the shown location?



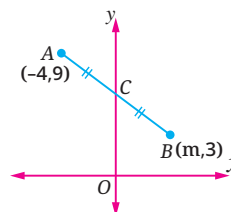
15 **Sports :**

The graph shows the positions of three players during a part of a water polo match. Player A throws the ball to player B who throws it to player C, then player C returns it to player A. Find the distances, in meters, covered by the ball assuming it moves in a straight line.

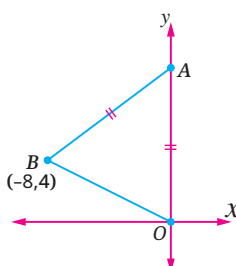
Creative Thinking



- 16 In the given figure:
Orthogonal coordinate system,
C is the midpoint of $\overline{AB}$
Find the numerical value of the expression:
 $AB + OC$



- 17 In the given figure:
Calculate the area of $\triangle AOB$



Evaluate your understanding!

How well do you understand The Distance between Two Points in The Coordinate Plane? Tick the right box



Lesson 3 - 7

Slope of the Straight Line

■ Learning Outcomes

- Understand the concept of slope of the straight line.
- Find the slope of the straight line in the coordinate system.
- Use the slope of the straight line to solve some real-life applications.

■ Vocabulary

- Slope
- The change of x
- The change of y
- Velocity

■ Think & Discuss

Can the formula :

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

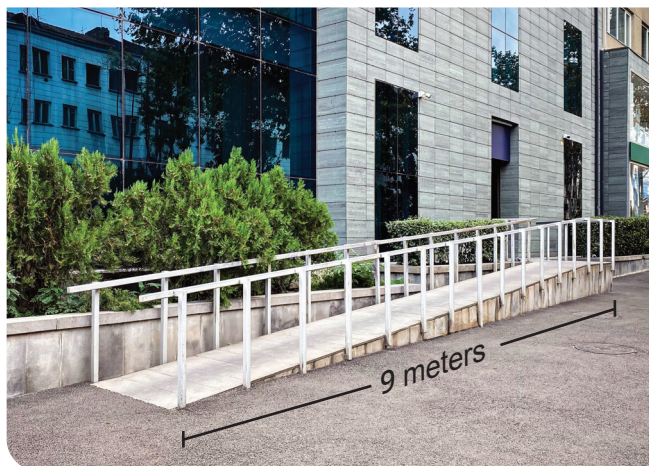
be used to find the slope (m) of the straight line passing through the two points

$(x_1, y_1), (x_2, y_2)$?

Get Ready!



The global standard used to measure the slope of wheelchair ramps is a maximum ratio of 1 : 12 , meaning that for every 1 meter of elevation a horizontal distance of 12 meters is needed. If the ramp in the picture was built according to this global standard, what is the elevation of the end of the ramp?

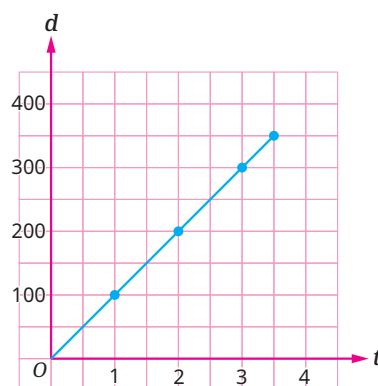


In this lesson, you will learn about the slope of the straight line and its applications in order to solve such real life problems.

Think & Discuss!



The given graph represents the relation between the distance (d) in kilometers and time (t) in hours for Sameh's trip by his car on one of the highways that allow a maximum speed of 120 km/hour. Did Sameh exceed the speed limit during his trip? Discuss.



Learn!

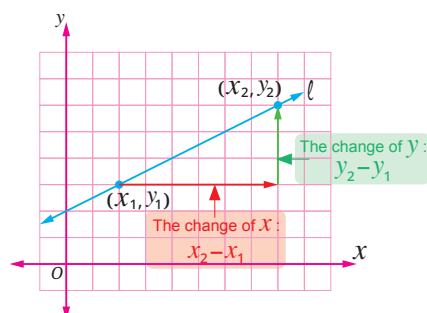


The slope of the straight line

The slope of the straight line is a measure of the steepness of the line, it is the ratio between the vertical change (the change of y) and the horizontal change (the change of x) between any two points on it in the coordinate plane, and it is usually denoted by the letter " m ".

The slope of the straight line passing through the two points $(x_1, y_1), (x_2, y_2)$ in the coordinate system is given by the relation:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



■ Self-Evaluation ①

In each of the following, find the slope of $\overleftrightarrow{AB}$:

- ① $A(5, 4), B(3, 1)$
- ② $A(3, -4), B(2, -1)$
- ③ $A(1, 2), B(7, 2)$
- ④ $A(-5, 3), B(-5, 4)$

■ Remember that

The fraction whose denominator is 0 is undefined.

Example 1

In each of the following find the slope of $\overleftrightarrow{AB}$:

- ① $A(1, 2), B(4, 5)$
- ② $A(-1, 2), B(3, 1)$
- ③ $A(-2, 5), B(1, 5)$
- ④ $A(-2, 1), B(-2, 5)$



①

Assuming that:

$(1, 2)$ is (x_1, y_1) ,

$(4, 5)$ is (x_2, y_2)

$$\therefore m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 2}{4 - 1} = \frac{3}{3} = 1$$

$\therefore$ The slope of $\overleftrightarrow{AB} = 1$

②

Assuming that:

$(-1, 2)$ is (x_1, y_1) ,

$(3, 1)$ is (x_2, y_2)

$$\therefore m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 2}{3 - (-1)} = -\frac{1}{4}$$

$\therefore$ The slope of $\overleftrightarrow{AB} = -\frac{1}{4}$

③

Assuming that:

$(-2, 5)$ is (x_1, y_1) ,

$(1, 5)$ is (x_2, y_2)

$$\therefore m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 5}{1 - (-2)} = \frac{0}{3} = 0$$

$\therefore$ The slope of $\overleftrightarrow{AB} = \text{zero}$

④

Assuming that:

$(-2, 1)$ is (x_1, y_1) ,

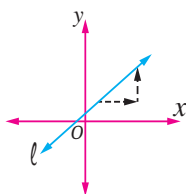
$(-2, 5)$ is (x_2, y_2)

$$\therefore m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 1}{-2 - (-2)} = \frac{4}{0}$$

$\therefore$ The slope of $\overleftrightarrow{AB}$ is undefined

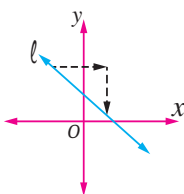
Note that:

Positive slope:



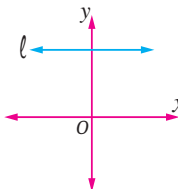
The value of y increases as x increases for every point moving on the straight line.

Negative slope:



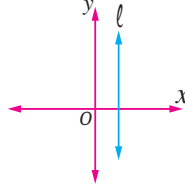
The value of y decreases as x increases for every point moving on the straight line.

The slope equals zero:



The value of y is constant for every point on the straight line.

The undefined slope:



The value of x is constant for every point on the straight line.

■ Self-Evaluation ②

- ① If the slope of the straight line passing through the two points $(7, a)$, $(6, 1)$ equals -6 , what is the value of a ?
- ② If the straight line passing through the two points $(k, 7)$, $(-3, 2)$ is parallel to the y -axis, find the value of k .

■ Self-Evaluation ③

Find the value of k that makes the points $A(-2, -5)$, $B(1, 1)$, $C(4, k)$ lie on one straight line.

Example 2

Find the value of k in each of the following if:

- ① The slope of the straight line passing through the two points $(-1, 5)$, $(7, k)$ equals $\frac{3}{2}$
- ② The straight line passing through the two points $(3, 7)$, $(5, k)$ is parallel to the X -axis.



①

$$\begin{aligned}\therefore m &= \frac{y_2 - y_1}{x_2 - x_1} \\ \therefore \frac{3}{2} &= \frac{k - 5}{7 - (-1)} \\ \therefore \frac{3}{2} &= \frac{k - 5}{8} \\ \therefore 2k - 10 &= 24 \\ \therefore 2k &= 34 & \therefore k &= 17\end{aligned}$$

②

$$\begin{aligned}\therefore \text{The straight line} &\parallel \text{The } X\text{-axis} \\ \therefore \text{The slope of the straight line} &= 0 \\ \therefore m &= \frac{y_2 - y_1}{x_2 - x_1} \\ \therefore \frac{7 - k}{3 - 5} &= 0 & \therefore \frac{7 - k}{-2} &= 0 \\ \therefore 7 - k &= 0 & \therefore k &= 7\end{aligned}$$

Example 3

Prove that the points $A(0, 1)$, $B(3, 3)$, $C(9, 7)$ lie on one straight line (collinear).



Assuming that: the slope of $\overrightarrow{AB} = m_1$, and the slope of $\overrightarrow{BC} = m_2$

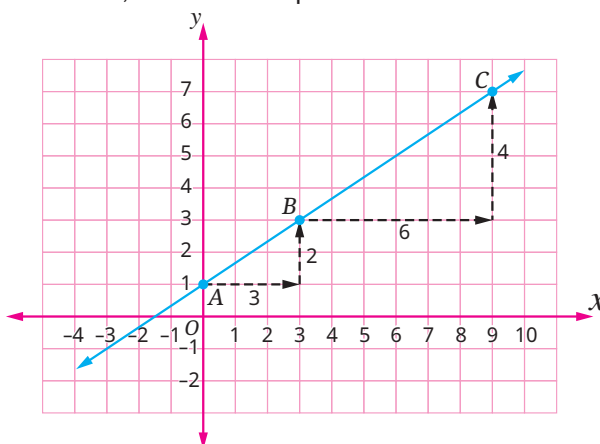
$$\begin{aligned}\therefore m_1 &= \frac{3 - 1}{3 - 0} = \frac{2}{3}, & m_2 &= \frac{7 - 3}{9 - 3} = \frac{4}{6} = \frac{2}{3} \\ \therefore m_1 &= m_2\end{aligned}$$

$\therefore$ The points A, B, C lie on one straight line.

Check the solution :

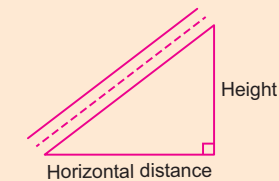
When representing the points A, B, C graphically you find that:

The ratio of the vertical change (the change of y) to the horizontal change (the change of x) is constant between every two points. Meaning that the slope is constant, therefore the points lie on the same straight line.



■ Road Gradient

The slope of the straight line is used as a measure for road steepness.



$$\text{Slope of the road} = \frac{\text{Height}}{\text{Horizontal distance}}$$

■ Self-Evaluation ④

A road rises at a rate of 2 meters for every 25 horizontal meters, what is the height of the end of the road if the length of the road is 300 meters?

■ Self-Evaluation ⑤

The following table expresses the relation between the amount of fuel consumed while a car is moving and the distance covered. Calculate the fuel consumption rate.

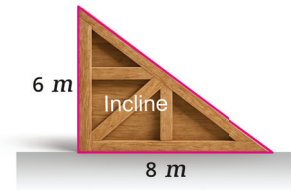
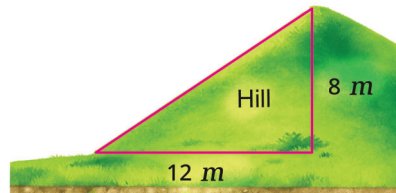
Distance covered in kilometers	100	200	300
Amount of fuel consumed in liters	10	20	30

Real life applications on slope of the straight line

The slope of the straight line is a ratio between the vertical change and the horizontal change for any two points on the straight line and it is a constant ratio expressing the rate of change of y with respect to x , so the slope of the straight line is used in many applications where we need to calculate the rate such as road gradient, velocity, growth rates, and fuel consumption.

Example 4

In the two following figures :



Which is more difficult to ascend: the hill or the incline ?



Assuming that: m_1 is the slope of the hill, m_2 is the slope of the incline

$$\therefore m_1 = \frac{8}{12} = \frac{2}{3}, \quad m_2 = \frac{6}{8} = \frac{3}{4}$$

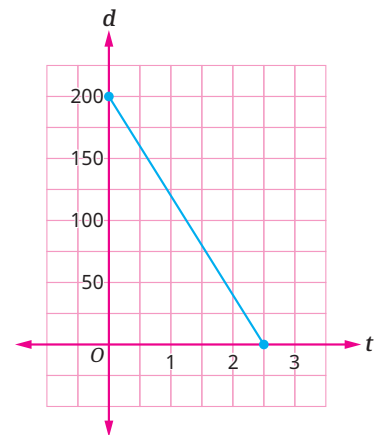
$$\therefore m_2 > m_1$$

$\therefore$ The incline is more difficult to ascend than the hill.

Example 5

The given graph expresses the relation between the distance (d), in kilometers, and time (t), in hours, for the return trip of Magdy's car from Cairo to Alexandria.

Find the magnitude of the uniform velocity of the car.



Assuming that: $(0, 200)$ is (t_1, d_1) and $(2.5, 0)$ is (t_2, d_2)

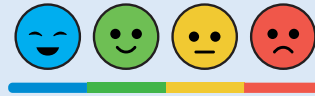
The slope of the straight line expresses the uniform velocity (V) of the car.

$$\therefore V = \frac{d_2 - d_1}{t_2 - t_1} = \frac{0 - 200}{2.5 - 0} = -80$$

$\therefore$ The magnitude of the car velocity = 80 km/hour

And the negative sign indicates that the trip is in the return direction.

Lesson Assessment



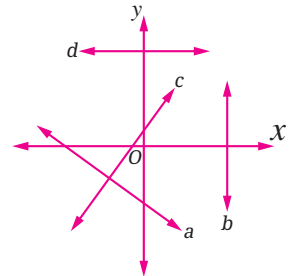
First

Measuring Conceptual Understanding



① In the given figure:

Classify the slope of each of the straight lines a , b , c , d
(Positive - Negative - Zero - Undefined).



► Choose the correct answer from the given ones:

② What is the slope of the straight line passing through the two points

$A(-2, 3)$, $B(5, 7)$?

- (a) $\frac{7}{4}$ (b) $\frac{4}{7}$ (c) $\frac{4}{3}$ (d) $\frac{2}{3}$

③ What is the slope of the straight line passing through the two points $A(3, 2)$, $B(4, 2)$?

- (a) 2 (b) 1 (c) 0 (d) Undefined

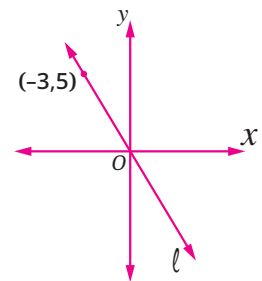
④ If the slope of the straight line passing through the two points $(a, 3)$, $(2, b)$ equals $\frac{1}{2}$ what is the value of a ?

- (a) $4 - 2b$ (b) $8 - 2b$ (c) $2b - 4$ (d) $2b - 8$

⑤ In the given figure:

What is the slope of the straight line ℓ ?

- (a) $\frac{5}{3}$ (b) $\frac{3}{5}$
(c) $-\frac{3}{5}$ (d) $-\frac{5}{3}$



Second

Applying Concepts

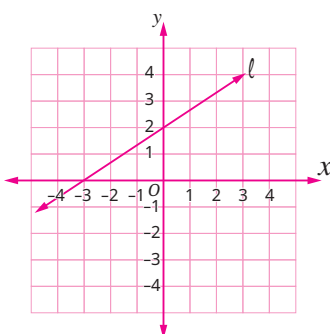


⑥ Find the slope of the straight line passing through each pair of points:

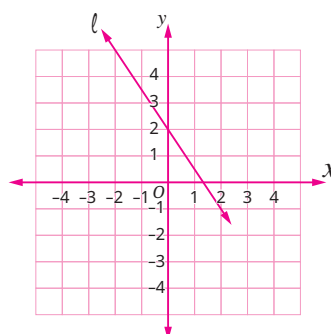
- ① $(2, 4)$, $(6, 3)$ ② $(1, -6)$, $(-3, 4)$ ③ $(-6, 0)$, $(6, 0)$
④ $(2, 6)$, $(2, 0)$ ⑤ $(6, 3)$, $(-5, -3)$ ⑥ $(4, -4)$, $(-2, 2)$

⑦ Find the slope of the straight line ℓ in each of the following:

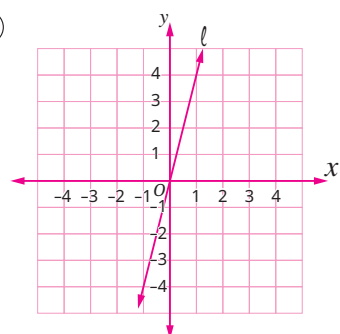
①



②



③



8 Find the value of k in each of the following :

① $A(-2, 5), B(3, k), m = 2$

② $A(2, 3), B(k, -6), m = -\frac{3}{2}$

③ $A(3, -2), B(-2, k), m = 0$

9 If the slope of the straight line passing through the two points $A(a+2, 3), B(-a, -9)$ equals 2, find the value of a

10 In each of the following, show whether the three given points are collinear or not.

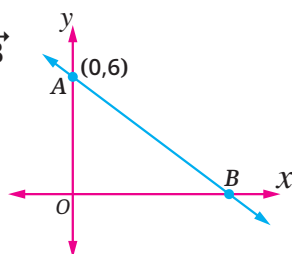
① $A(2, 1), B(-4, 4), C(6, -1)$

② $D(5, -5), E(7, -11), F(-7, -1)$

11 In the following figure:

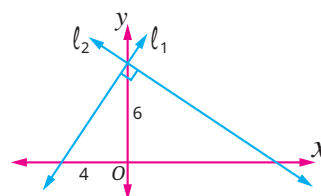
If the area of the triangle $OAB = 24$ square units,

Find slope of $\overleftrightarrow{AB}$



12 In the following figure:

Find the slope of each of the two straight lines l_1, l_2 .

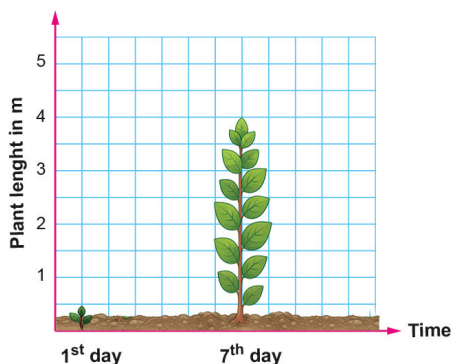


Third

Analysis and Subjects Integration



13 **Mountain Climbing:** Omar climbs a rocky hill that rises 12 meters for every 6 horizontal meters. And Khaled climbs another hill that rises 25 meters for every 12 horizontal meters. Which of the two hills is steeper?



14 **Plants:**

An initial measurement of the length of a climbing plant was made and it was 0.5 meter, and after seven days the length of the plant became 4 meters.

① Draw a graph representing the relation between plant length and time.

② What is the slope of the line in the graph? And what does this slope represent?

③ Assuming that the plant's growth rate continues at the same pace, what will be the length of the plant after 15 days?

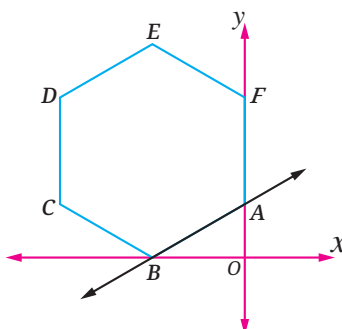
Creative Thinking



15 In the given figure:

$ABCDEF$ is a regular hexagon

Find the slope of $\overleftrightarrow{AB}$



Evaluate your understanding!

How well do you understand The Slope of The Straight Line? Tick the right box



Unit Three Assessment

Evaluate your understanding!

How well do you understand Unit Three?
Tick the right box



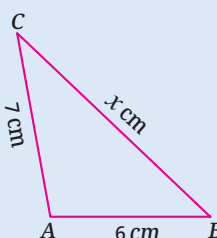
► Choose the correct answer from the given ones:

- ① If the slope of the straight line passing through the two points $(-4, k)$, $(6, -7)$ equals $-\frac{1}{3}$, what is the value of k ?
(a) 23 (b) -23 (c) $\frac{11}{3}$ (d) $-\frac{11}{3}$

- ③ In the given figure:

What are the possible values of X that make $m(\angle A) > m(\angle B)$?

- (a) $6 < X < 7$
(b) $1 < X < 13$
(c) $7 < X < 13$
(d) $X > 7$



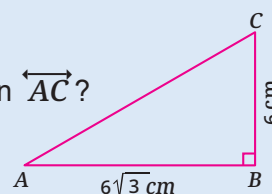
- ② What is the area of the circle whose circumference equals 14π cm?

- (a) 7π cm² (b) 14π cm²
(c) 49π cm² (d) 196π cm²

- ④ In the given figure:

What is the length of the projection of $\overline{AB}$ on $\overleftrightarrow{AC}$?

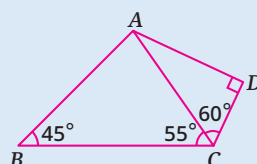
- (a) 12 cm
(b) 9 cm
(c) 6 cm
(d) 3 cm



► Complete each of the following with the correct answer:

- ⑤ In the given figure:

The ascending order of the five drawn line segments according to their lengths is

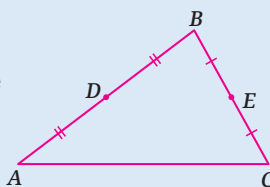


- ⑦ In the given figure:

If $D(-1, -2)$, $E(2, 3)$ are midpoints of $\overline{AB}$, $\overline{BC}$

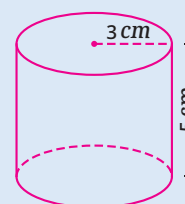
Then the length of

$\overline{AC}$ = units of length.



- ⑥ In the given figure:

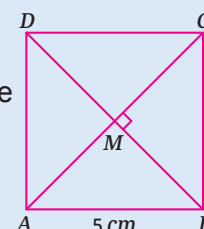
The surface area of the cylinder equals cm²
($\pi \approx 3.14$)



- ⑧ In the given figure:

If $ABCD$ is a square, then the length of projection of $\overline{AC}$ on

$\overleftrightarrow{BD}$ = cm.



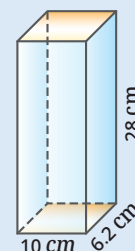
► Answer the following questions:

- ⑨ The points in the given table lie on one straight line.
Find the slope of the straight line and the value of k .

x	-3	2	7	k
y	0	2	4	6

- ⑩ In the given figure:

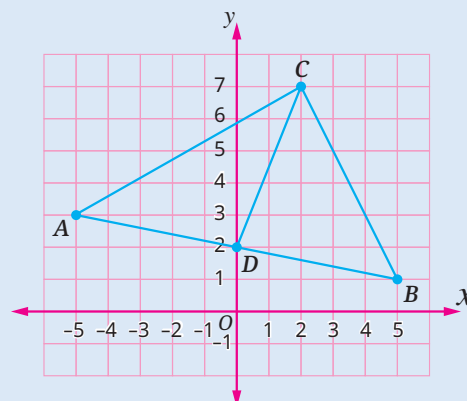
The height of a right prism is 28 cm, if the height increased by 0.7 cm,
What is the amount of increase in the surface area of the prism?



11 In the given figure:

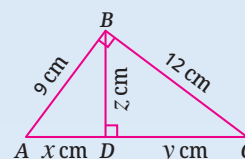
Prove that:

$$m(\angle ADC) > m(\angle BDC)$$



12 In the given figure:

Find the values of x , y , z



Unit Three Activity The circle in our life

► Activity Goal:

Employing concepts and skills of geometry in real-life situations.

► Implementation Steps:

Divide the class into groups each group consisting of (4 - 5) students, and ask them to implement the following steps:

- 1 Determine (3) shapes and solids in their surrounding (such as: wall clock - plate - ...) which contain models of a circle.
- 2 Determine the appropriate tool according to this shape / solid to measure the radius length or diameter length.
- 3 Calculate the circumference and area of the circle in each shape / solid.
- 4 Complete the following table:

Shape Name	Measuring Tool	Diameter or radius length	The circumference value	The area value
1.				
2.				
3.				

- 5 Write a paragraph explaining the level of accuracy in the measurement process and calculation of the data shown in the table.



UNIT 4

Statistics and Probability

Unit Lessons

4-1 Samples

4-2 Probability - Prediction of Future Events



The concept of sample and its selection is one of the foundations on which the availability of data that artificial intelligence deals with, while theoretical probability defines the mathematical rules that the model uses in predicting and extracting results, the better the sample with correct probability calculations, the more it helps artificial intelligence to reach more accurate results.

► Can methods and ways of selecting samples be developed to help artificial intelligence obtain more accurate results?

Issues and Life Skills

- Mathematical Communication
- Critical Thinking
- Decision Making
- Predicting

Values

- Solidarity
- Food Security
- Sports Awareness

Lesson 4 - 1

Samples

■ Learning Outcomes

- Understand sources of data collection.
- Understand methods of data collection.
- Understand the concept of the sample.
- Understand types of samples and ways of selecting them.
- Determine the best method to select a sample.

■ Vocabulary

- Biased selection
- Random selection
- Simple random sample
- Layer random sample

Get Ready!



Every year, the Ballon d'or is awarded to the best football player in the world. Can you imagine how difficult it would be if every fan in the world was asked to vote to choose the winning player?!

For this reason, the opinion of all football fans is not polled, but a small group of sports journalists, analysts, and experts from different countries is chosen to represent the opinions of fans in the world.

This small group is called a sample, and it represents the large population consisting of millions of fans. If you want to choose the best player in your school this year, how can you choose a sample of students that expresses everyone's opinion?

In this lesson, you will learn the concept of sample, its types, and how to select samples; which will enable you to solve such problems.



Think & Discuss!



A student wanted to know the favorite sport of students in his school, so he asked only the basketball team students. The result he reached was that "Basketball is the favorite sport for all students in the school". Is this conclusion accurate? Discuss.



Learn!



Data Collection

- The statistical researcher collects, organizes, and analyzes data to reach accurate results that are used in making correct decisions.
- Data collection requires knowing its sources and determining the method of collecting it.

■ Think with your classmates

Which of the following statistical data sources are primary and which are secondary?

- 1 Searching on internet sites for the results of the Egyptian National Football Team in the previous African Nations Championship.
- 2 Polling your classmates' opinions about the name of their favorite subject.

■ What is the Statistical Population?

The statistical population is all the individuals (items) that the researcher is interested in studying and which are gathered by one general characteristics. For example: Students of a certain school represent a statistical population whose item is "the student".

■ Think

Why do we use the sample method to study the following populations:

- 1 Infinite populations such as the population of (desert sands)?
- 2 Examining a patient's blood?

Sources of data collection

1 Primary sources:

They are sources from which we obtain data directly through personal interview, questionnaire, or observation. They are characterized by accuracy, but they need time, effort, and large cost.

2 Secondary sources:

They are sources from which we obtain data that has been collected and recorded before by others or institutions, such as: bulletins of the Central Agency for Public Mobilization and Statistics, the Internet, employee database, ... These sources saving time, effort, and money.



Methods of data collection

The method used to collect data depends on the goal for which this data is collected, as well as on the size of the statistical population. And from these methods of collecting data the following methods:

1 Mass population method:

This method is based on collecting data from all individuals of the statistical population and is used in surveying all individuals of the population, such as: "General Population Census", "Elections", etc. It is distinguished by accuracy of results, inclusiveness, and lack of bias, but it needs a long time, great effort, and high cost.

2 Sample method:

This method is based on collecting data from a sample representing the population and conducting research on it, then the results are generalized to the whole population.

For examples: "A sample of a patient's blood to conduct some medical tests", "A sample of some products of a factory to research their conformity to specifications", etc.

This method saves time, effort, and costs, but it gives inaccurate results especially

if the sample does not represent the population accurately and is called in this case a biased sample.



Concept of sample

It is a small part of a large population that resembles the population and represents it.

Types of samples according to method of selection

First Biased selection (Non-random sample)

It means selecting some specific individuals from the population to suit the research objectives.

For example: When studying the extent of satisfaction of a school's students with a school trip, the sample must be chosen from the students who attended this trip neglecting the rest, and this selection is not considered random.

Second Random selection (Random sample)

It means selecting the sample such that every individual of the population individuals has the same chance of selection. From the most important types of random samples the following :

1 Simple random sample:

It is used with homogeneous populations, and is selected in two ways according to the number of population individuals as follows:

First method: (If the population size is small)

This method is done as follows:

- A card is prepared for each individual of the population on which the name or number of the individual is written, such that all cards are identical in terms of color and size.
- The cards are folded identically, placed in a box, then mixed well.
- A card is drawn from the box randomly, then the cards are stirred well and we choose a second card, and so on until we select the required number for the sample.



This method is considered suitable for selecting a sample consisting of 5 students from a class having 40 students.

Second method: (If the population size is large)

This method is done as follows:

- All individuals of the population are numbered.
- The random number in the scientific calculator is used, by pressing the following keys in order.



A decimal number appears between 0.000, 0.999
And in case one decimal digit appears after the decimal point we add two zeros to make it a part of a thousand ($0.3 \rightarrow 0.300$),

And in case two decimal digits appear after the decimal point we add a zero on the right to make it a part of a thousand ($0.82 \rightarrow 0.820$), then we take those numbers after ignoring the decimal point and choose the individual representing the population.

- With repeated pressing of the key $\boxed{=}$ numbers appear successively until we reach the sample number we want, and numbers greater than the study population number are excluded, as well as numbers selected before are also excluded.

This method is considered suitable for selecting a sample consisting of 40 students from a school having 800 students.



■ Note that

The most appropriate sample size is on average 10% of the population size.

■ Self-Evaluation ①

If you want to take a sample from your school to do research on practicing sports by students on a daily basis or not, A random sample is taken from the school the size of the sample is 10% of the total school number.

Show how this sample can be chosen using the calculator knowing that the total number of the school is 500 students.

Example 1

A company has 300 workers and it is required to know the workers' opinion on the health insurance system of the company through a questionnaire prepared for this purpose. If this questionnaire is for a random sample whose size is 10% of the total number of workers in this company, clarify how this sample is chosen using the calculator.



∴ The number of workers in the company = 300 workers.

∴ The number of the random sample = $\frac{10}{100} \times 300 = 30$ workers.

And the 30 workers are chosen as follows:

- Each worker of the company is given a number from 1 to 300.
- The calculator is used to choose 30 workers by the previously mentioned method which is confined between 0, 300. And random numbers appearing greater than 300 are excluded, and repeated numbers are also excluded.

For example: By pressing the keys:



If we get the decimal fraction 0.162 the chosen worker number is 162.

And if we get the decimal fraction 0.24 the chosen worker number is 240.

And if we get the decimal fraction 0.511 it is excluded, because number 511 is greater than 300 and so on until we get 30 numbers.

2 Layer random sample:

It is used with non-homogeneous populations, by dividing the population into qualitative groups differing in characteristics, and it is done as follows:

- The statistical population individuals are divided into layers according to their characteristics.
- The researcher chooses a random sample to represent each layer according to its size in the population, and it is called the layer random sample.

For example: To study school studying level of a population consisting of 450 students from the first and second preparatory, If we know that the ratio between the number of first grade students to second is $\frac{4}{5}$, and we want to choose a sample consisting of 54 students, we have to choose 24 students from the first grade, 30 students from the second grade randomly.

■ Self-Evaluation ②

A factory produced 400 refrigerators of the "16 foot size", 300 refrigerators of "18 foot size", 100 refrigerators of "20 foot size".

If we wanted to draw a layer sample consisting of 72 refrigerators such that it is representative of all produced types to examine them, calculate the number of individuals of each layer in the sample.

Example ②

A school has 400 boys, 500 girls, It is required to do a questionnaire on a layer sample consisting of 45 boys and girls in which each layer is represented according to its size. Calculate the number of individuals of each layer in the sample.



The total number of students = $400 + 500 = 900$ boys and girls.

The number of boys in the sample

$$= \frac{400}{900} \times 45 = 20 \text{ boys.}$$

The number of girls in the sample

$$= \frac{500}{900} \times 45 = 25 \text{ girls.}$$

Lesson Assessment



First

Measuring Conceptual Understanding



► Choose the correct answer from the given ones:

- ① The sample method is considered the appropriate method for each of the following, except
 - (a) Examining factory production
 - (b) Examining desert sands
 - (c) Knowing mass population
 - (d) Examining a patient's blood
- ② A hospital has 40 doctors, 200 nurses, a layer sample of size 36 individuals was taken in which each layer is represented according to its size, then the number of nurses in this sample equals nurses?
 - (a) 6
 - (b) 18
 - (c) 24
 - (d) 30

Second

Applying Concepts



- ③ A school having 350 students conducted a questionnaire to know which of the products sold in the school canteen is best for students (hot drink - pizza - iced juices - hot meal) and a number was given to each student, then a random sample was chosen to conduct this questionnaire whose size is 10% of the school number. Determine using the calculator the students' numbers in this sample
- ④ In a factory, there are 180 workers, 120 technicians, 80 engineers, a layer sample of size 57 people was taken, find the number of individuals of each layer in the sample
- ⑤ It is required to take a layer random sample in which each layer is represented according to its size from a population consisting of 5000 individuals and divided into four layers, such that the number of individuals of the first layer was 1500 individuals, and the number of individuals of the first layer in the sample was 225 individuals, find the sample size

Evaluate your understanding!

How well do you understand samples?

Tick the right box



Lesson 4 - 2

Probability - Prediction of Future Events

■ Learning Outcomes

- Remember the concept of theoretical probability.
- Use probability laws to predict the occurrence of future events.

■ Vocabulary

- Probability
- Prediction of future events

Get Ready!



The Egyptian Meteorological Authority follows the weather condition daily and issues its forecasts regarding various weather phenomena including rainfall. If the Meteorological Authority announced that the probability of rainfall in Alexandria on a day of November is 0.4, how many days is it expected that rain will fall in that month?



Stanley Bridge is one of the landmarks of Alexandria city

In this lesson, you will learn the concept of probability and predict future events; which will enable you to solve such real life problems.

Think & Discuss!



Mazen is training on taking penalty kicks in football. Mazen took 5 kicks and scored 3 goals. If Mazen took 20 kicks, what is the expected number of goals Mazen will score? Discuss.

Learn!



Probability and prediction of future events

Probability helps in predicting the occurrence of future events based on the information we have, it gives us an idea about the possibility of occurrence of each event, and is used in our daily life in many fields, such as: Meteorology, Economics, Sports games and competitions, Scientific research, etc.

Previous knowledge: theoretical probability

Theoretical Probability is based on the principle of equal opportunities or equal possibilities, and equals the ratio between the number of event outcomes and the total number of outcomes.

Meaning that:

$$\text{Probability of occurrence of any event (A)} = \frac{\text{Number of outcomes of event A}}{\text{Total number of all outcomes}}$$

And the probability of occurrence of event A is denoted by $P(A)$.

■ Self-Evaluation ①

On one day, the numbers of milk packs sold in one of the food store chains were as in the following table:

Milk Type	Number of packs sold
Full Cream	100
Half Cream	25
Skimmed	75

And it is expected that the total number of sold packs will increase tomorrow to reach 600 packs, so how many packs of skimmed type are expected to be sold tomorrow?

■ Remember that

Probability can be written in the form of a common fraction or a decimal fraction or a percentage.

■ Self-Evaluation ②

A factory produces two types of shoes. To conduct a study to adjust the production quantity according to the market requirements, a random sample was chosen from sales of 5 factory outlets, the size of each is 80 shoes, and its data was distributed as follows:

Outlet Number	Type 1 Sales	Type 2 Sales
1	45	35
2	19	61
3	27	53
4	36	44
5	39	41

- Which of the two types is most in demand? And what do you advise the factory?
- If the total production of this factory is 4,800 shoes, what is the expected number to produce from the first type?

Example 1

When taking a sample consisting of 30 students out of 750 students in one school and asking them about their favorite activity, their answers were as in the table:

Activity	Frequency
Football	15
Swimming	8
Handball	4
Volleyball	3

- If a student was chosen randomly, what is the probability that the student is among those preferring swimming activity?
- What is the expected number of students who prefer swimming activity in the school?
- What is the expected number of students who prefer volleyball activity in the school?



- The probability that the student is among those preferring swimming activity = $\frac{8}{30} = \frac{4}{15}$
- The expected number of students who prefer swimming activity in the school = $\frac{4}{15} \times 750 = 200$ students.
- $\therefore$ The probability that the student is among those preferring volleyball activity = $\frac{3}{30} = \frac{1}{10}$
 $\therefore$ The expected number of students who prefer volleyball activity in the school = $\frac{1}{10} \times 750 = 75$ students.

Example 2

A mobile phone factory produces two types (A1), (A2) of phones. To conduct a study to adjust the production quantity according to market requirements, a random sample was chosen from sales of 5 factory outlets, the size of each is 60 phones, and the data was distributed as follows:



Outlet number	1	2	3	4	5
Sales of type (A1)	35	48	24	19	45
Sales of type (A2)	25	12	36	41	15

- Which of the two types is most in demand? And what do you advise the factory?
- If the total production of this factory is 5,000 phones, what is the expected number to produce from the first type according to this study?

■ Self-Evaluation ③

A random sample was chosen from applicants for a job test consisting of 60 persons, it was found that 45 persons are fluent in English, 30 persons are fluent in French, 25 persons are fluent in both languages together.

- ① If one of the applicants was chosen randomly, what is the probability that the chosen person is not fluent in either of the two languages?
- ② If the total number of applicants for the test is 2,400 persons, find the expected number of persons who are fluent in English only.

■ Note that

Number of those preferring Math is 60 students, number of those preferring Math and Social Studies together is 35 students.

∴ The number of those preferring Math only:
 $60 - 35 = 25$

Similarly, number of those preferring Social Studies only:
 $45 - 35 = 10$

∴ The number of those not preferring either of the two subjects:

$$80 - (10 + 25 + 35) = 10$$



- ① The total sales in the five outlets of type (A1)

$$= 35 + 48 + 24 + 19 + 45 = 171 \text{ phones.}$$

The total sales in the five outlets of type (A2)

$$= 25 + 12 + 36 + 41 + 15 = 129 \text{ phones.}$$

∴ Type(A1) is the most in demand, and we advise the factory to increase production of this type.

- ② Probability of sales from type (A1) =

number of sold phones from type (A1)

The total number of sold phones from both types

$$= \frac{171}{300} = 0.57$$

∴ The number expected to be produced from Type (A1)
 $= 0.57 \times 5,000 = 2,850 \text{ phones.}$

Example ③

A random sample consisting of 80 students in one of the schools was included in a survey, it was found that 60 students of them prefer Mathematics, 45 students of them prefer Social Studies, 35 students prefer both subjects. If a student was chosen randomly:



First: Calculate the probability of each of the two following events:

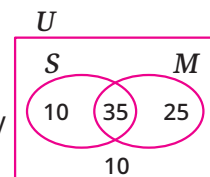
- ① Event A "The chosen student prefers at least one of the two subjects".
- ② Event B "The chosen student prefers only one of the two subjects".

Second: If the total number in the school is 2,000 students:

- ① What is the expected number of students who prefer Social Studies only?
- ② What is the expected number of students who prefer at most one subject of Mathematics and Social Studies?



Assuming that M is the set of students preferring Mathematics, S is the set of students preferring Social Studies. The numbers of students included in the survey can be illustrated using the opposite Venn diagram.



First:

- ① The probability of (A) = $\frac{\text{Number of students prefer one of the two subjects at least}}{\text{The total number of students in the sample}}$

$$\therefore P(A) = \frac{25 + 35 + 10}{80} = \frac{70}{80} = \frac{7}{8}$$

- ② The probability of (B) = $\frac{\text{Number of students prefer one of the two subjects only}}{\text{The total number of students in the sample}}$

$$\therefore P(B) = \frac{25 + 10}{80} = \frac{35}{80} = \frac{7}{16}$$

■ Self-Evaluation ④

A random sample consisting of 25 students was chosen from students applying for a Mathematics competition in one of the preparatory schools, and the students were distributed as in the following table:

	First Grade	Second Grade	Total
Male	8		
Female		4	10
Total			

First: Complete the previous table.

Second: Find the probability that the first place winner is:

- ① Male from the first grade.
- ② Female.
- ③ From the second grade.
- ④ Male or from the first grade.

Third: If the total number of students in this competition is 1550 students, what is the expected number of second grade female students?

Second:

① The probability that the student prefers Social Studies only = $\frac{10}{80} = \frac{1}{8}$

∴ The expected number of school students preferring Social

Studies only = $\frac{1}{8} \times 2,000 = 250$ students.

② ∴ The probability that the student prefers at most one subject of Mathematics and Social Studies = $\frac{25 + 10 + 10}{80} = \frac{45}{80} = \frac{9}{16}$

∴ The expected number of school students preferring at most one subject of Mathematics and Social Studies

= $\frac{9}{16} \times 2,000 = 1,125$ students.

Example ④

A random sample consisting of 60 visitors of a museum distributed as in the table:

First: Complete the table.

Second: If one of the visitors was chosen randomly, calculate the probability of each of the following events:

	Arab	Foreigner	Total
Male	24		32
Female	16	12	
Total			

- ① Event A "The chosen person is female".
- ② Event B "The chosen person is foreigner".
- ③ Event C "The chosen person is foreigner male".
- ④ Event D "The chosen person is male or foreigner".

Third: If the total number of museum visitors during the last month equals 2250 visitors, what is the expected number of foreigner males in this month?



First:

	Arab	Foreigner	Total
Male	24	8	32
Female	16	12	28
Total	40	20	60

Second: ① $P(A) = \frac{28}{60} = \frac{7}{15}$

② $P(B) = \frac{20}{60} = \frac{1}{3}$

③ $P(C) = \frac{8}{60} = \frac{2}{15}$

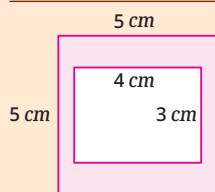
④ $P(D) = \frac{24 + 8 + 12}{60} = \frac{44}{60} = \frac{11}{15}$

Third:

∴ The probability that the chosen person is foreigner male = $\frac{2}{15}$

∴ The expected number of foreigner males = $\frac{2}{15} \times 2,250 = 300$ visitors.

■ Self-Evaluation ⑤

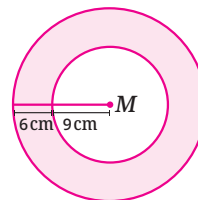


If a person shoots an arrow at this board and hit it:

- ① Find the probability of hitting the colored region.
- ② If 75 persons shoot an arrow each, what is the expected number of times the colored region will be hit?

Example 5

In a sports day at school, students are asked to shoot at the opposite board. If all students hit the board:



- ① What is the probability that the student hits the colored part of the board?
- ② If 300 students played this game, what is the expected number of students who can hit the colored part of the board?



- ① The probability of hitting the colored part = $\frac{\text{Area of the colored part}}{\text{The total area}}$

$$\therefore \text{The area of the large circle} = \pi (15)^2 = 225 \pi \text{ cm}^2$$

$$\text{The area of the small circle} = \pi (9)^2 = 81 \pi \text{ cm}^2$$

$$\text{The area of the colored part} = 225 \pi - 81 \pi = 144 \pi \text{ cm}^2$$

$$\therefore \text{The probability of hitting the colored part} = \frac{144 \pi}{225 \pi} = \frac{16}{25}$$

- ② The expected number of students who can hit the colored part = $300 \times \frac{16}{25} = 192$ students.

Lesson Assessment



First

Measuring Conceptual Understanding



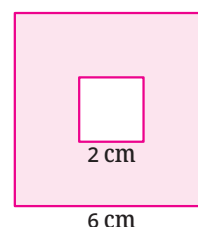
► Choose the correct answer from the given ones:

- ① If the probability of a football team winning in a single match is 0.75, how many matches is expected to win out of a total of 24 matches?
(a) 6 (b) 12 (c) 18 (d) 20
- ② A factory produces 800 bulbs daily. If the probability that the bulb is defective is 0.03, then the expected number of working bulbs produced in one day is bulbs.
(a) 24 (b) 124 (c) 560 (d) 776
- ③ The probability of obtaining a good rice crop in a village is $\frac{3}{5}$, and farmers planted X feddans of rice, If the expected number of feddans to produce a good crop is 30 feddans, what is the value of X ?
(a) 18 (b) 36 (c) 42 (d) 50
- ④ In a mixed school with 1,200 students, a random sample of 90 students was chosen, and it was found that the number of girls is 54, what is the expected number of girls in the school?
(a) 480 girls (b) 660 girls (c) 720 girls (d) 840 girls
- ⑤ If the school media team consists of 18 members, including 4 teachers, 6 female students, 8 male students, If one of the team members was chosen randomly to give a speech, what is the probability that it is a male student or a teacher?
(a) $\frac{2}{9}$ (b) $\frac{1}{3}$ (c) $\frac{4}{9}$ (d) $\frac{2}{3}$



- ⑥ In front of you is a board in the form of two squares, if a person shoot an arrow and hits the board.

- ① Find the probability of hitting the colored region.
- ② If 90 persons shoot an arrow each and hit the board, what is the expected number of times the colored region will be hit ?



- ⑦ A class has some students wearing glasses, and others do not wear glasses. If a student was chosen randomly from this class, and the probability that this student wears glasses is 0.1, find the probability that this student does not wear glasses, and if the number of students in this class is 40 students, find the expected number of students who wear glasses.



- ⑧ A student conducted a questionnaire on a sample consisting of 25 students from his school students to know which sports they prefer to play and recorded the results in the given table:

Sport	Football	Basketball	Volleyball	Total
Number of students		4	2	25

- ① If one of the school students was chosen randomly, find the probability that he prefers playing football.
 - ② Find the expected number of students who prefer playing football out of the 600 students in the school.
- ⑨ A class has 36 students, 27 students passed in Mathematics, 12 students passed in Arabic, 9 students passed in both exams. If a student was chosen randomly

First: Find the probability that the chosen student:

- ① passed in Mathematics.
- ② passed in Arabic.
- ③ failed in Arabic.
- ④ failed in both Mathematics and Arabic.

Second: If the total number of the school is 540 students, what is the expected number of students passing in Mathematics only?

- ⑩ Two factories for producing televisions produce 210 televisions daily and some of these televisions are defective according to the given table.

First: Complete the given table.

Second: If one television was chosen randomly, calculate the probabilities of the following events:

- ① Event A "The chosen television is defective"
- ② Event B "The chosen television is produced in the second factory".
- ③ Event C "The chosen television is defective and is produced in the second factory".
- ④ Event D "The chosen television is defective or is produced in the second factory".

Third: If the production of the two factories in the next month will reach 7,000 televisions, what is the expected number of non-defective televisions in that month?

	First Factory Production	Second Factory Production	Total
Defective	12		30
Non-defective			
Total	100		210



Third

Analysis and Subjects Integration



- 11 On a certain day, 60 students in a school watched videos on a platform, 35 of them watched educational videos, 25 watched entertainment videos, 10 watched both types. What is the probability that a student is chosen randomly and has watched an educational video only? What is the probability that the student has watched an educational or entertainment video? And if there are 2,400 students in the school, what is the expected number of students who watched educational videos?



Adhering to traffic signals and prescribed speeds is civilized behavior that preserves everyone's safety

- 12 A car insurance company pays an amount of 3,500 pounds as compensation for a car exposed to an accident. If the probability of car accident is 0.006 and the number of subscribers in this policy is 9,000 subscribers, what is your expectation of the compensations the company bears?

- 13 Last year, a country's exports of onions were 96,280 tons and 82,220 tons of potatoes and 11,500 tons of grapes. If it is expected that the total exports of this country in the current year of onions, potatoes, and grapes will increase to reach 380,000 tons, how many tons of grapes will be exported this year?



Creative Thinking



- 14 The following table shows the results of a test designed to give positive result if the patient is infected with a virus or give negative result if the patient is not infected with the virus.

	Negative Result	Positive Result	Total
Patient infected with virus	14		200
Patient not infected with virus		46	
Total			480

Complete the previous table, then find the probability that the test gives false results

Evaluate your understanding!

How well do you understand Probability - Prediction of Future Events?

Tick the right box



Unit Four Assessment

Evaluate your understanding!

How well do you understand Unit Four?
Tick the right box

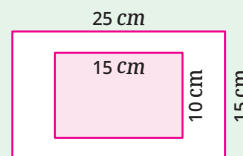


► Choose the correct answer from the given ones:

- ① There are 320 workers and engineers in a factory, a random sample consisting of 16 workers and engineers was chosen from them, it was found that the number of engineers in the sample is 5, what is the expected number of engineers in the factory?
(a) 50 (b) 100
(c) 160 (d) 220
- ② Which of the following is a secondary source for collecting data?
(a) Personal interview
(b) Questionnaires
(c) Employee database
(d) Observation and measurement
- ③ If the probability of scoring a goal from a penalty throw in a handball match is 60%, How many goals are expected to be scored out of 30 throws ?
(a) 9 (b) 12
(c) 18 (d) 30
- ④ The mass population method is considered suitable for which of the following?
(a) Researching desert sand components.
(b) Examining the water quality of a well.
(c) Researching the percentage of presence of a mineral in mining areas.
(d) Knowing the number of students scored full marks in the Mathematics exam in the class.

► Complete each of the following:

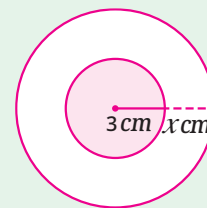
- ⑤ A factory produces 120 washing machines daily. If the probability that the washing machine is defective is 0.05, then the expected number of working washing machines produced in one day is washing machines.
- ⑥ If the medical team in a hospital consists of 60 nurses, and 12 doctors, and one of the team members was chosen randomly, then the probability that this member is a nurse equals
- ⑦ Among methods of data collection ,
- ⑧ In the given figure:
If a person shoot at the given board,
Then the probability of hitting the colored region equals



► Answer the following questions:

- ⑨ A random sample consisting of 50 students participated in a survey, there are 25 students prefer the Arabic, 20 students prefer Science, 15 students prefer both Arabic and Science. If a student was chosen randomly find the probability that the chosen student prefers at least one of the two subjects, and if the number of students in the school is 750, what is the expected number of students who prefer Science only?

- 10 In the given figure :
If the probability of hitting the colored region equals $\frac{1}{9}$
what is the value of x ?



- 11 A random sample of a school students consisting of 24 boys and 16 girls. If there are 9 boys, and 4 girls wear glasses and a person was chosen randomly from this school find the probability that this person is a boy who does not wear glasses, and if the total number of students in the school is 440 then, find the expected number of boys who do not wear glasses in the school.
- 12 A car factory produces 3 models of cars in the year. The number of cars produced of the first model is 500, from the second model 700, from the third model 300. If the factory management decided to take a 7% sample of the total production and each model is represented according to its production size, find the number of cars of each model in the sample.

Unit Four Activity

List of Predictions or Future Activities

- **Activity Goal:** Possessing the skill of employing probability concepts and prediction of future events in life situations.

► **Implementation Steps:**

Divide the class into groups each group consisting of (4 – 5) students, and ask them to study the following situation, then implement the steps shown:

The school wishes to implement one of the following activities, and is seems help from the students to predict the best future choice for students:

- Sports activities (Basketball - Football - Volleyball).
- Holding competitions (Cultural - Scientific - Sports)
- Organizing trips (Alexandrina library - The grand Egyptian museum - The administrative capital).

► **Student Tasks:**

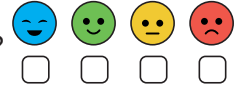
- 1 Each group has to choose only one topic from the three previous topics.
- 2 Determine the appropriate data collection method for this topic.
- 3 Prepare the appropriate tool for data collection, and determine the sample on which it will be applied.
- 4 Use the tool and start collecting and organizing data.
- 5 Represent the data using one of the graphical representations learned before.
- 6 Provide the appropriate recommendation to the school in light of the prediction reached from those results.



Final Assessment

Evaluate your understanding!

How well do you understand the lessons of second preparatory?
Tick the right box



First Group

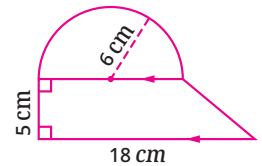
► Choose the correct answer from the given ones:

- ① If $X, 5, 12, 30$ are proportional, what is the value of X ?
(a) 2 (b) 3 (c) 4 (d) 16
- ② What is the solution set of the inequality $-X > 4$ in R ?
(a) $]-\infty, -4[$ (b) $]-\infty, 4[$
(c) $]-4, \infty[$ (d) $]4, \infty[$
- ③ If $f(X) = X + 3$, $g(X) = X^2$, what is the value of $2f(3) + 3g(2)$?
(a) 12 (b) 18 (c) 24 (d) 36
- ④ If the probability of a team winning any match is 80 % How many matches is the team expected to win out of a total of 35 matches?
(a) 7 (b) 14 (c) 21 (d) 28
- ⑤ The circumference of the base of a right circular cylinder is 16π cm and its height is 2 cm, what is its volume?
(a) 16π cm³ (b) 32π cm³
(c) 64π cm³ (d) 128π cm³
- ⑥ What is the slope of the straight line passing through the two points $(2, 7), (-4, 8)$?
(a) $-\frac{1}{2}$ (b) $-\frac{1}{6}$ (c) $-\frac{2}{3}$ (d) -6
- ⑦ What is the distance between the two points $(4, 7), (1, 11)$?
(a) 5 length units (b) 6 length units
(c) 7 length units (d) 8 length units
- ⑧ The diameter length of a circle is 12 cm. What is the circumference of this circle?
(a) 6π cm (b) 12π cm
(c) 18π cm (d) 24π cm
- ⑨ The base of a right prism is in the shape of a square with side length 5 cm, and its height is 6 cm, what is its lateral area?
(a) 30 cm² (b) 120 cm²
(c) 150 cm² (d) 170 cm²

Second Group

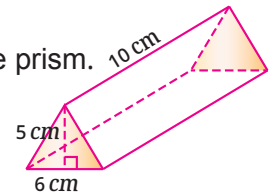
► Answer the following questions:

- ① If b is a mean proportional between a, c , prove that: $\frac{a-b}{b-c} = \frac{a+3b}{3c+b}$
- ② Represent graphically the function f where $f(X) = 4 - 2X$
- ③ Find the area of the figure, knowing that: $(\pi \approx \frac{22}{7})$



- ④ In the given figure:

Find the volume of the prism.



- ⑤ If R is a relation from X to Y where, $X = \{2, 3, 5\}$, $Y = \{5, 7, 8\}$ and " $X R y$ " means " $X + y = 10$ " for every $X \in X$, $y \in Y$ Write R as a set of ordered pairs, and represent it using an arrow diagram and a graph. Does the relation represent a function or not? why?

- ⑥ In the given figure:

$ABCD$ is a quadrilateral in which

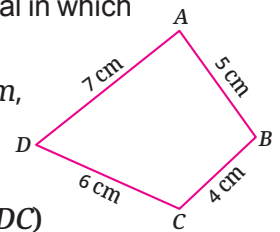
$AB = 5$ cm,

$BC = 4$ cm, $CD = 6$ cm,

$DA = 7$ cm

Prove that:

$m(\angle ABC) > m(\angle ADC)$

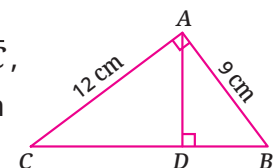


- ⑦ In the given figure:

If $\overline{AB} \perp \overline{AC}$, $\overline{AD} \perp \overline{BC}$,

find the length of each

of $\overline{AD}$, $\overline{CD}$



المقاس	وزن المتن	ألوان المتن	وزن الغلاف	ألوان الغلاف
19 × 27 سم	70 جم ورق أبيض	4 لون	180 جم كوشيه	4 لون